Ch 5/6 Free Response Questions

1.

Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

(a) Write an equation for y, the amount of oil remaining in the well at any time t.

(b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

(c) In order not to lose money, at what time t should oil no longer be pumped from the well?

2.

(a) Find the general solution of the differential equation $xy' + y = 0$.

(b) Find the general solution of the differential equation $xy' + y = 2x^2y$.

(c) Find the particular solution of the differential equation in part (b) that satisfies the condition that $y = e^2$ when $x = 1$.

3.

Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$.

4.

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

5.

The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

6.

Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

7.

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

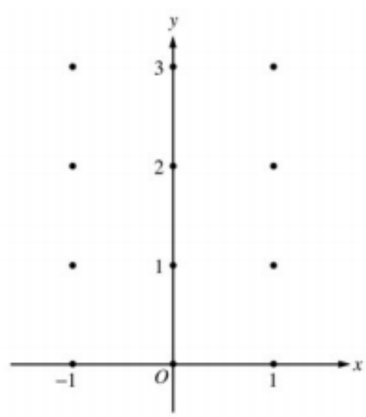
8.

Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



9.

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

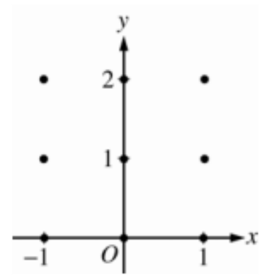
(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

10.

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.



(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

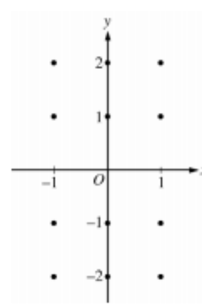
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

11.

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.



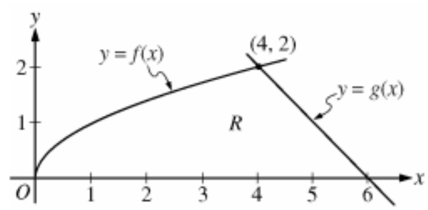
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

12.



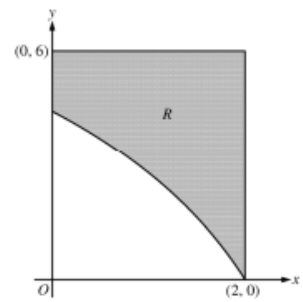
The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

13. CALCULATOR



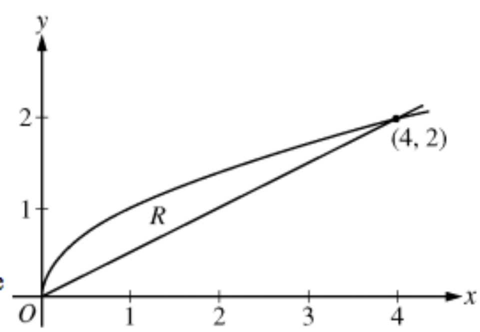
In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

14.



Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

15.

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

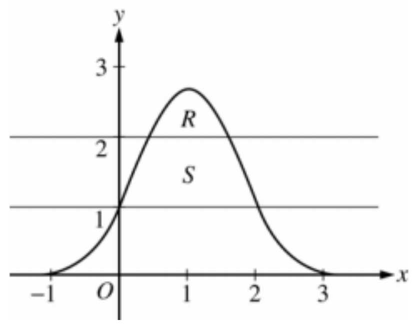
16. CALCULATOR

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



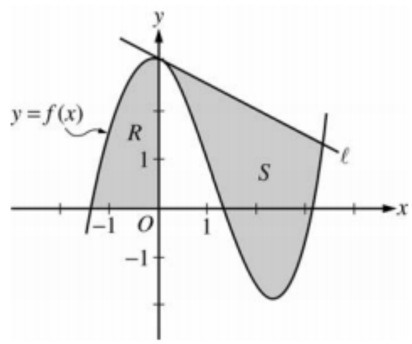
17. CALCULATOR

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.



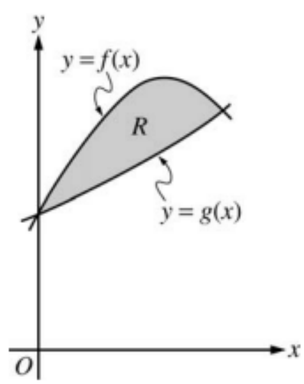
18. CALCULATOR

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

(a) Find the area of R.

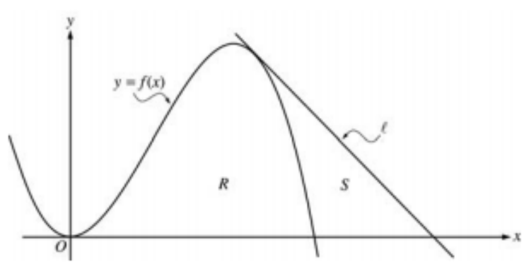
(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.



19. CALCULATOR

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.



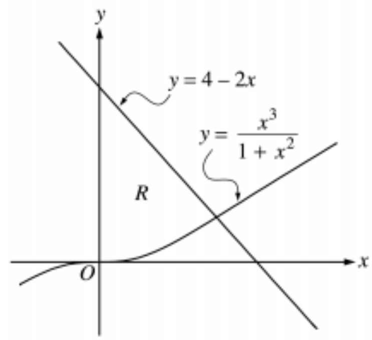
(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

20. CALCULATOR

Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

21. NO CALCULATOR (OMIT part (b))

Let R be the region enclosed by the graph of $y = \dfrac{x^2}{x^2+1}$, the line $x = 1$, and the x-axis.

(a) Find the area of R .

(b) Find the volume of the solid generated when R is rotated about the <u>y-axis</u>.

22. NO CALCULATOR (OMIT part (c))

Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

(a) Find the area of R .

(b) Find the volume of the solid generated when R is revolved about the <u>x-axis</u>.

(c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the <u>y-axis</u>.

ANSWERS

1.

(a) $$\frac{dy}{dt} = ky \qquad y = Ce^{kt}$$ or $$\begin{cases} \dfrac{dy}{y} = k\,dt \\ \ln|y| = kt + C_1 \\ y = e^{kt+C_1} \end{cases}$$

$$t = 0 \Rightarrow C = 10^6,\ C_1 = \ln 10^6$$
$$\therefore y = 10^6 e^{kt}$$
$$t = 6 \Rightarrow \frac{1}{2} = e^{6k}$$
$$\therefore k = -\frac{\ln 2}{6}$$
$$y = 10^6 e^{\frac{-t}{6}\ln 2} = 10^6 \cdot 2^{\frac{-t}{6}}$$

(b) $$\frac{dy}{dt} = ky = -\frac{\ln 2}{6} \cdot 6 \cdot 10^5$$
$$= -10^5 \ln 2$$

Decreasing at $10^5 \ln 2$ gal/year

(c) $$5 \cdot 10^4 = 10^6 e^{kt}$$
$$\therefore kt = -\ln 20$$
$$\therefore t = \frac{-\ln 20}{\frac{-\ln 2}{6}}$$
$$= 6\frac{\ln 20}{\ln 2} = 6\log_2 20$$

$6\dfrac{\ln 20}{\ln 2}$ years after starting

2.

a) $y = A/x$

b) $y = Ae^{x^2 - \ln x}$

c) $y = \dfrac{e^{x^2+1}}{x}$

3.

(a) $$\frac{dy}{dx} = \frac{3x^2+1}{2y}$$
$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=4}} = \frac{3+1}{2\cdot 4} = \frac{4}{8} = \frac{1}{2}$$

(b) $$y - 4 = \frac{1}{2}(x-1)$$
$$f(1.2) - 4 \approx \frac{1}{2}(1.2-1)$$
$$f(1.2) \approx 0.1 + 4 = 4.1$$

(c) $$2y\,dy = (3x^2+1)\,dx$$
$$\int 2y\,dy = \int (3x^2+1)\,dx$$
$$y^2 = x^3 + x + C$$
$$4^2 = 1 + 1 + C$$
$$14 = C$$
$$y^2 = x^3 + x + 14$$

$y = \sqrt{x^3 + x + 14}$ is branch with point $(1,4)$

$$f(x) = \sqrt{x^3 + x + 14}$$

(d) $$f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$$

4.

(a) $$e^{2y}\,dy = 3x^2\,dx$$
$$\frac{1}{2}e^{2y} = x^3 + C_1$$
$$e^{2y} = 2x^3 + C$$
$$y = \frac{1}{2}\ln(2x^3 + C)$$
$$\frac{1}{2} = \frac{1}{2}\ln(0 + C); \quad C = e$$
$$y = \frac{1}{2}\ln(2x^3 + e)$$

(b) Domain: $2x^3 + e > 0$
$$x^3 > -\frac{1}{2}e$$
$$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$$

Range: $-\infty < y < \infty$

5.

(a) $\dfrac{d^2y}{dx^2} = 2y\dfrac{dy}{dx}(6-2x) - 2y^2$

$= 2y^3(6-2x)^2 - 2y^2$

$\left.\dfrac{d^2y}{dx^2}\right|_{\left(3,\frac{1}{4}\right)} = 0 - 2\left(\dfrac{1}{4}\right)^2 = -\dfrac{1}{8}$

(b) $\dfrac{1}{y^2}\,dy = (6-2x)\,dx$

$-\dfrac{1}{y} = 6x - x^2 + C$

$-4 = 18 - 9 + C = 9 + C$

$C = -13$

$y = \dfrac{1}{x^2 - 6x + 13}$

6.

(a) $\dfrac{dy}{dx} = 0$ when $x = 3$

$\left.\dfrac{d^2y}{dx^2}\right|_{(3,-2)} = \left.\dfrac{-y - y'(3-x)}{y^2}\right|_{(3,-2)} = \dfrac{1}{2}$,

so f has a local minimum at this point.

or

Because f is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which $y < 0$. On this interval, $\dfrac{dy}{dx}$ is negative to the left of $x = 3$ and $\dfrac{dy}{dx}$ is positive to the right of $x = 3$. Therefore f has a local minimum at $x = 3$.

(b) $y\,dy = (3-x)\,dx$

$\dfrac{1}{2}y^2 = 3x - \dfrac{1}{2}x^2 + C$

$8 = 18 - 18 + C$; $C = 8$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

7.

(a) $f''(x) = \sqrt{f(x)} + x\cdot\dfrac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \dfrac{x^2}{2}$

$f''(3) = \sqrt{25} + \dfrac{9}{2} = \dfrac{19}{2}$

(b) $\dfrac{1}{\sqrt{y}}\,dy = x\,dx$

$2\sqrt{y} = \dfrac{1}{2}x^2 + C$

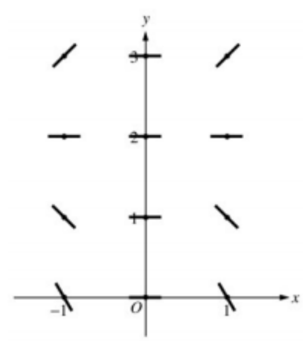
$2\sqrt{25} = \dfrac{1}{2}(3)^2 + C$; $C = \dfrac{11}{2}$

$\sqrt{y} = \dfrac{1}{4}x^2 + \dfrac{11}{4}$

$y = \left(\dfrac{1}{4}x^2 + \dfrac{11}{4}\right)^2 = \dfrac{1}{16}\left(x^2 + 11\right)^2$

8.

(a)



(b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

(c) $\dfrac{1}{y-2}\,dy = x^4 dx$

$\ln|y-2| = \dfrac{1}{5}x^5 + C$

$|y-2| = e^C e^{\frac{1}{5}x^5}$

$y - 2 = Ke^{\frac{1}{5}x^5}$, $K = \pm e^C$

$-2 = Ke^0 = K$

$y = 2 - 2e^{\frac{1}{5}x^5}$

9.

(a) $2yy' = y + xy'$

$(2y - x)y' = y$

$y' = \dfrac{y}{2y - x}$

(b) $\dfrac{y}{2y - x} = \dfrac{1}{2}$

$2y = 2y - x$

$x = 0$

$y = \pm\sqrt{2}$

$(0, \sqrt{2}), (0, -\sqrt{2})$

(c) $\dfrac{y}{2y - x} = 0$

$y = 0$

The curve has no horizontal tangent since $0^2 \neq 2 + x \cdot 0$ for any x.

(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \dfrac{7}{3}$.

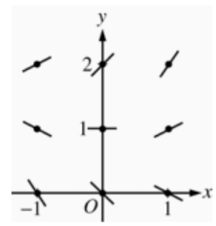
$\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt} = \dfrac{y}{2y - x} \cdot \dfrac{dx}{dt}$

At $t = 5$, $6 = \dfrac{3}{6 - \frac{7}{3}} \cdot \dfrac{dx}{dt} = \dfrac{9}{11} \cdot \dfrac{dx}{dt}$

$\left.\dfrac{dx}{dt}\right|_{t=5} = \dfrac{22}{3}$

10.

(a)



(b) $\dfrac{d^2y}{dx^2} = \dfrac{1}{2} + \dfrac{dy}{dx} = \dfrac{1}{2}x + y - \dfrac{1}{2}$

Solution curves will be concave up on the half-plane above the line $y = -\dfrac{1}{2}x + \dfrac{1}{2}$.

(c) $\left.\dfrac{dy}{dx}\right|_{(0,\,1)} = 0 + 1 - 1 = 0$ and $\left.\dfrac{d^2y}{dx^2}\right|_{(0,\,1)} = 0 + 1 - \dfrac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

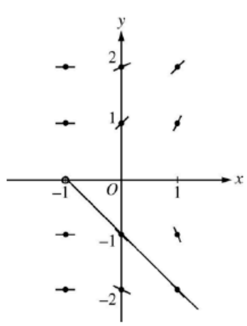
(d) Substituting $y = mx + b$ into the differential equation:

$m = \dfrac{1}{2}x + (mx + b) - 1 = \left(m + \dfrac{1}{2}\right)x + (b - 1)$

Then $0 = m + \dfrac{1}{2}$ and $m = b - 1$: $m = -\dfrac{1}{2}$ and $b = \dfrac{1}{2}$.

11.

(a)



(b) $-1 = \dfrac{x+1}{y} \Rightarrow y = -x - 1$

$\dfrac{dy}{dx} = -1$ for all (x, y) with $y = -x - 1$ and $y \neq 0$

(c) $\int y\, dy = \int (x + 1)\, dx$

$\dfrac{y^2}{2} = \dfrac{x^2}{2} + x + C$

$\dfrac{(-2)^2}{2} = \dfrac{0^2}{2} + 0 + C \Rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

Since the solution goes through $(0, -2)$, y must be negative. Therefore $y = -\sqrt{x^2 + 2x + 4}$.

12.

(a) Area $= \int_0^4 \sqrt{x}\, dx + \dfrac{1}{2} \cdot 2 \cdot 2 = \left.\dfrac{2}{3}x^{3/2}\right|_{x=0}^{x=4} + 2 = \dfrac{22}{3}$

(b) $y = \sqrt{x} \Rightarrow x = y^2$

$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$\dfrac{1}{2\sqrt{x}} = 1 \Rightarrow x = \dfrac{1}{4}$

The point P has coordinates $\left(\dfrac{1}{4}, \dfrac{1}{2}\right)$.

13.

(a) $\int_0^2 (6 - 4\ln(3 - x))\, dx = 6.816$ or 6.817

(b) $\pi\int_0^2 \left((8 - 4\ln(3 - x))^2 - (8 - 6)^2\right) dx$
$= 168.179$ or 168.180

(c) $\int_0^2 (6 - 4\ln(3 - x))^2\, dx = 26.266$ or 26.267

14.

(a) Area $= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big|_{x=0}^{x=4} = \frac{4}{3}$

(b) Volume $= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4}\right) dx$

$= \frac{x^2}{2} - \frac{2x^{5/2}}{5} + \frac{x^3}{12}\Big|_{x=0}^{x=4} = \frac{8}{15}$

(c) Volume $= \pi\int_0^4 \left(\left(2 - \frac{x}{2}\right)^2 - (2 - \sqrt{x})^2\right) dx$

15.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

(b) $\pi\int_0^3 \left((3y + 1)^2 - \left(y^2 + 1\right)^2\right) dy$

$= \frac{207\pi}{5} = 130.061$ or 130.062

(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

16.

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$
Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx -$ Area of R
$= 2.06016 -$ Area of $R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$
$= 0.219064 + 1.107886 + 0.219064 = 1.546$

(c) Volume $= \pi\int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2 - 1)^2\right) dx$

17.

For $x < 0$, $f(x) = 0$ when $x = -1.37312$.
Let $P = -1.37312$.

(a) Area of $R = \int_P^0 f(x)\,dx = 2.903$

(b) Volume $= \pi\int_P^0 \left((f(x)+2)^2 - 4\right) dx = 59.361$

(c) The equation of the tangent line ℓ is $y = 3 - \frac{1}{2}x$.

The graph of f and line ℓ intersect at $A = 3.38987$.

Area of $S = \int_0^A \left(\left(3 - \frac{1}{2}x\right) - f(x)\right) dx$

18.

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.

(a) Area $= \int_0^S (f(x) - g(x))\,dx$

$= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx$

$= 0.429$

(b) Volume $= \pi\int_0^S \left((f(x))^2 - (g(x))^2\right) dx$

$= \pi\int_0^S \left((1+\sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx$

$= 4.266$ or 4.267

(c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x)-g(x)}{2}\right)^2 dx$

$= \int_0^S \frac{\pi}{2}\left(\frac{1+\sin(2x)-e^{x/2}}{2}\right)^2 dx$

$= 0.077$ or 0.078

19.

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$

$f(3) = 36 - 27 = 9$

Tangent line at $x = 3$ is

$y = -3(x-3) + 9 = -3x + 18$,

which is the equation of line ℓ.

(b) $f(x) = 0$ at $x = 4$

The line intersects the x-axis at $x = 6$.

Area $= \frac{1}{2}(3)(9) - \int_3^4 \left(4x^2 - x^3\right) dx$

$= 7.916$ or 7.917

OR

Area $= \int_3^4 \left((18 - 3x) - \left(4x^2 - x^3\right)\right) dx$

$+ \frac{1}{2}(2)(18 - 12)$

$= 7.916$ or 7.917

(c) Volume $= \pi\int_0^4 \left(4x^2 - x^3\right)^2 dx$

$= 156.038\pi$ or 490.208

20.

Region R

$\frac{x^3}{1+x^2} = 4 - 2x$ at $x = 1.487664 = A$

(a) Area $= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right) dx$

$= 3.214$ or 3.215

(b) Volume

$= \pi\int_0^A \left[(4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right] dx$

$= 31.884$ or 31.885 or 10.149π

(c) Volume $= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right)^2 dx$

$= 8.997$

21.	22.
(a) $\text{Area} = \int_0^1 \frac{x^2}{x^2+1}\,dx$ $= \int_0^1 1 - \frac{1}{x^2+1}\,dx$ $= x - \arctan x \Big\|_0^1$ $= 1 - \frac{\pi}{4}$ (b) $\text{Volume} = 2\pi \int_0^1 x\left(\frac{x^2}{x^2+1}\right)dx$ $= 2\pi \int_0^1 x - \frac{x}{x^2+1}\,dx$ $= 2\pi \left(\frac{x^2}{2} - \frac{1}{2}\ln\left\|x^2+1\right\|\right)\Big\|_0^1$ $= \pi\left(1 - \ln 2\right)$	a) (a) $A = \int_0^1 e^x - (x-1)^2\,dx$ $= \int_0^1 e^x - x^2 + 2x - 1\,dx$ $= e^x\Big]_0^1 - \frac{1}{3}(x-1)^3\Big]_0^1$ $= (e-1) - \frac{1}{3} = e - \frac{4}{3}$ b) $\pi\int_0^1 (e^{2x} - (x-1)^4)dx = \pi\left(\frac{e^2}{2} - \frac{7}{10}\right)$