Chapter 4 Free Response

1. No Calculator

A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

(a) Write an equation for the velocity $v(t)$ of the particle.

(b) Write an equation for the position $x(t)$ of the particle.

(c) For what values of t, $0 \leq t \leq \pi$, is the particle at rest?

2. No Calculator

A particle moves on the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of t for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

3. No Calculator

A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

(b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

4. No Calculator

A particle moves along the x-axis with acceleration given by $a(t) = \cos t$ for $t \geq 0$. At $t = 0$, the velocity $v(t)$ of the particle is 2, and the position $x(t)$ is 5.

(a) Write an expression for the velocity $v(t)$ of the particle.

(b) Write an expression for the position $x(t)$.

(c) For what values of t is the particle moving to the right? Justify your answer.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

5. No Calculator

A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

(a) Find the minimum acceleration of the particle.

(b) Find the total distance traveled by the particle.

(c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

6. No Calculator

A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \ge 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

(a) Find the values of t for which the particle is at rest.

(b) Write an expression for the position $x(t)$ of the particle at any time $t \ge 0$.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

7. No Calculator

A particle starts at the point $(5,0)$ at $t = 0$ and moves along the x-axis in such a way that at time $t > 0$ its velocity $v(t)$ is given by $v(t) = \dfrac{t}{1+t^2}$.

(a) Determine the maximum velocity attained by the particle. Justify your answer.

(b) Determine the position of the particle at $t = 6$.

(c) Find the limiting value of the velocity as t increases without bound.

8. No Calculator

A particle moves along the x-axis in such a way that its acceleration at time t for $t > 0$ is given by $a(t) = \dfrac{3}{t^2}$. When $t = 1$, the position of the particle is 6 and the velocity is 2.

(a) Write an equation for the velocity, $v(t)$, of the particle for all $t > 0$.

(b) Write an equation for the position, $x(t)$, of the particle for all $t > 0$.

(c) Find the position of the particle when $t = e$.

9. No Calculator

A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

(a) Write an expression for the acceleration of the particle.

(b) For what values of t is the particle moving to the right?

(c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

10. No Calculator

The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

(a) Write an expression for the velocity v, in terms of time t, if $v = 5$ when $t = 0$.

(b) During the time that the velocity increases from 5 to 15, how far does the particle travel?

(c) Write an expression for the position s, in terms of time t, of the particle if $s = 0$ when $t = 0$.

11. No Calculaltor (Skip d)

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

12. Calculator

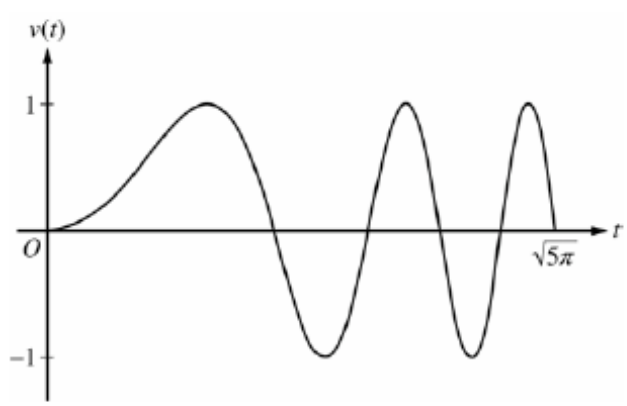
A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

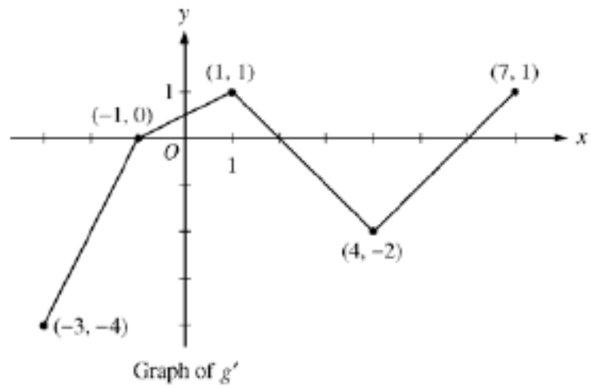
(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.



13. No Calculator

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

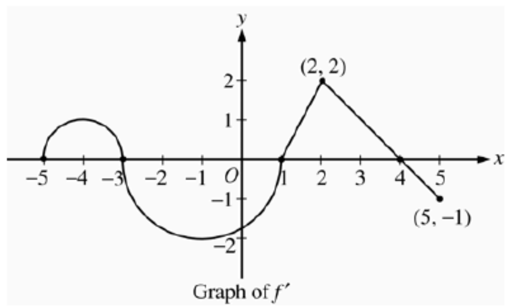
(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

14. No Calculator

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

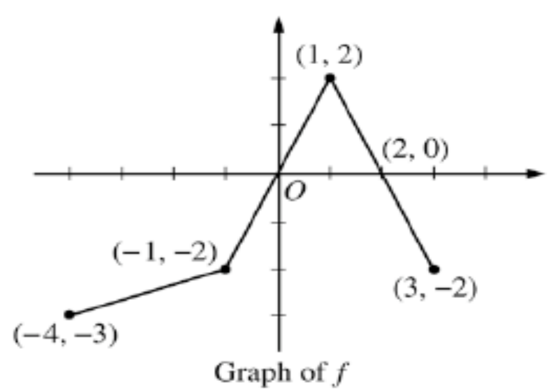
(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

15. No Calculator

The graph of the function f above consists of three line segments.



Graph of f

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

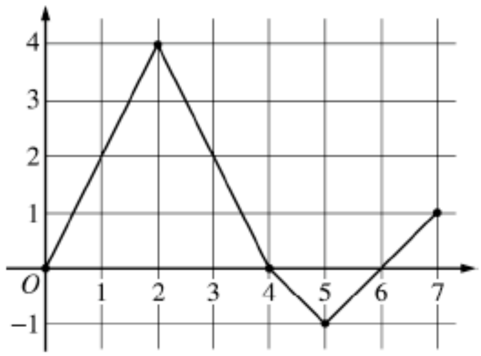
For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

16. No Calculator



Graph of f

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\, dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

17. No Calculator

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

18. No Calculator

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

19. Calculator

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

20. Calculator

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

21. No Calculator

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

22. Calculator

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

Chapter 4 Free Response Answers

1.

(a) $v(t)=\int 4\cos 2t\,dt$

$v(t)=2\sin 2t+C$

$v(0)=1 \Rightarrow C=1$

$v(t)=2\sin 2t+1$

(b) $x(t)=\int 2\sin 2t+1\;dt$

$x(t)=-\cos 2t+t+C$

$x(0)=0 \Rightarrow C=1$

$x(t)=-\cos 2t+t+1$

(c) $2\sin 2t+1=0$

$\sin 2t=-\frac{1}{2}$

$2t=\frac{7\pi}{6},\frac{11\pi}{6}$

$t=\frac{7\pi}{12},\frac{11\pi}{12}$

3.

(a) $x(t)=\int v(t)\,dt=\int(3t^2-2t-1)\,dt$

$=t^3-t^2-t+C$

$x(2)=8-4-2+C=5;\quad C=3$

$x(t)=t^3-t^2-t+3$

(b) avg. vel. $=\frac{x(3)-x(0)}{3-0}$

$=\frac{18-3}{3}=5$

$3t^2-2t-1=5$

$t=\frac{1+\sqrt{19}}{3}$ or 1.786

c)

$v(t)=3t^2-2t-1=0$

$t=-\frac{1}{3},\ t=1$

$x(0)=3$

$x(1)=1-1-1+3=2$

$x(3)=27-9-3+3=18$

distance $=(3-2)+(18-2)=17$

2.

(a) $x(t)=4t^3-18t^2+15t+C$

$0=x(1)=4-18+15+C$

Therefore $C=-1$

$x(t)=4t^3-18t^2+15t-1$

(b) $0=v(t)=12t^2-36t+15$

$3(2t-1)(2t-5)=0$

$t=\frac{1}{2},\ \frac{5}{2}$

(c) $\frac{dv}{dt}=24t-36$

$\frac{dv}{dt}=0$ when $t=\frac{3}{2}$

$v(0)=15$

$v\left(\frac{3}{2}\right)=-12$

$v(2)=-9$

Maximum velocity is 15

(d) Total distance $=\int_0^{1/2}v(t)\,dt-\int_{1/2}^{2}v(t)\,dt$

$=\left(x\left(\frac{1}{2}\right)-x(0)\right)-\left(x(2)-x\left(\frac{1}{2}\right)\right)$

$=\frac{5}{2}-(-1)-\left(-11-\frac{5}{2}\right)=17$

4.

(a) $v(t)=\sin(t)+C$

$2=\sin(0)+C$

$C=2$

$v(t)=\sin(t)+2$

(b) $x(t)=-\cos(t)+2t+C$

$5=-\cos(0)+2(0)+C$

$C=6$

$x(t)=-\cos(t)+2t+6$

(c) The particle moves to the right when $v(t)>0$,. i.e. when $\sin(t)+2>0$. This is true for all $t\geq 0$ because

$-1\leq\sin(t)\leq 1 \Rightarrow 0<-1+2\leq\sin(t)+2\leq 1+2$ for all t.

(d) The particle never changes directions since it moves to the right for all $t\geq 0$.

$x(0)=-\cos(0)+2(0)+6=5$

$x\left(\frac{\pi}{2}\right)=-\cos\left(\frac{\pi}{2}\right)+2\left(\frac{\pi}{2}\right)+6=\pi+6$

Distance $=x\left(\frac{\pi}{2}\right)-x(0)=\pi+1$

or

Distance $=\int_0^{\pi/2}|v(t)|\,dt=\int_0^{\pi/2}|\sin(t)+2|\,dt$

$=\int_0^{\pi/2}(\sin(t)+2)\,dt=(-\cos t+2t)\Big|_0^{\pi/2}=\pi+1$

5.

(a) $v(t)=3t^2-12t+9$

$a(t)=6t-12$

a is increasing, so a is minimum at $t=0$

$a(0)=-12$ is minimum value of a.

(b) Method 1:

$v(t)$	0 to 1	1 to 3	3 to 5
	$+$	$-$	$+$

$$d=\int_0^1(3t^2-12t+9)\,dt-\int_1^3(3t^2-12t+9)\,dt+\int_3^5(3t^2-12t+9)\,dt$$

$$=\left[t^3-6t^2+9t\right]_0^1-\left[t^3-6t^2+9t\right]_1^3+\left[t^3-6t^2+9t\right]_3^5$$

$$=4-(-4)+20=28$$

or

Method 2: $x(t)=t^3-6t^2+9t-2$

$\left[\text{or } x(t)=t^3-6t^2+9t+C\right]$

$x(0)=-2$

$x(1)=\ 2$

$x(3)=-2$

$x(5)=\ 18$

Total distance $=4+4+20=28$

(c) Method 1: $\dfrac{\int_0^5(3t^2-12t+9)\,dt}{5-0}$

$$=\frac{1}{5}\left[t^3-6t^2+9t\right]_0^5=\frac{1}{5}(20)=4$$

or

Method 2: $\dfrac{x(5)-x(0)}{5-0}=\dfrac{18-(-2)}{5}=4$

6.

(a) $v(t)=4t^3-4t$

$v(t)=4t^3-4t=0$

$=4t\left(t^2-1\right)=0$

Therefore $t=0,\ t=1$

(b) $x(t)=t^4-2t^2+C$

$3=x(1)=1^4-2\cdot1+C$

$3=C-1$

$4=C$

$x(t)=t^4-2t^2+4$

(c) $x(0)=4$

$x(1)=\ 3$

$x(2)=12$

Distance $=1+9=10$

7.

(a) The velocity has a maximum value of ½.

(b) $s(6)=5+\ln\sqrt{37}$

(c) 0

8.

(a) $v(t)=\int a(t)\,dt=\int \frac{3}{t^2}\,dt=-\frac{3}{t}+C$

$2=v(1)=-3+C$

Therefore $C=5$ and so $v(t)=-\frac{3}{t}+5$.

(b) $x(t)=\int v(t)\,dt=\int (-\frac{3}{t}+5)dt=-3\ln t+5t+C$

$6=x(1)=-3\ln 1+5+C$

Therefore $C=1$ and so $x(t)=-3\ln t+5t+1$.

(c) $x(e)=-3\ln e+5e+1=-3+5e+1=5e-2$

9.

(a) $a(t)=v'(t)=\ln t+t\cdot\frac{1}{t}-1=\ln t$

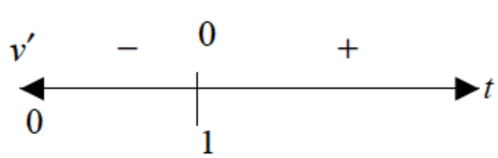
(b) $v(t)=t\ln t-t>0$

$t(\ln t-1)>0$

$t>e$

(c) $v'(t)=\ln t=0$

$t=1$



minimum velocity is $v(1)=-1$

10.

a. $v=5e^{2t}$

b. 5

c. $s=\frac{5}{2}e^{2t}-\frac{5}{2}$

11.

(a) $a(t) = v'(t) = -e^{1-t}$

$a(3) = -e^{-2}$

(b) $a(3) < 0$

$v(3) = -1 + e^{-2} < 0$

Speed is increasing since $v(3) < 0$ and $a(3) < 0$.

(c) $v(t) = 0$ when $1 = e^{1-t}$, so $t = 1$.

$v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.

Therefore, the particle changes direction at $t = 1$.

12.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

(b) Distance $= \int_0^3 |v(t)|\,dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and
$t = \sqrt{2\pi} = 2.50663$
$x(0) = 5$

$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t)\,dt = 5.89483$

$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t)\,dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t)\,dt = 5.77356$

$\left|x(\sqrt{\pi}) - x(0)\right| + \left|x(\sqrt{2\pi}) - x(\sqrt{\pi})\right| + \left|x(3) - x(\sqrt{2\pi})\right| = 1.702$

(c) $x(3) = 5 + \int_0^3 v(t)\,dt = 5.773$ or 5.774

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ $(t = 1.772, 3.070, 3.963)$.

Using $x(T) = 5 + \int_0^T v(t)\,dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

13.

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

(c) $\dfrac{g(7) - g(-3)}{7 - (-3)} = \dfrac{\frac{3}{2} - \frac{15}{2}}{10} = -\dfrac{3}{5}$

(d) $\dfrac{g'(7) - g'(-3)}{7 - (-3)} = \dfrac{1 - (-4)}{10} = \dfrac{1}{2}$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

14.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$
$$f(1) = 3$$
$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

15.

(a) $g(-1) = \int_{-4}^{-1} f(t)\,dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$
$g'(-1) = f(-1) = -2$
$g''(-1)$ does not exist because f is not differentiable at $x = -1$.

(b) $x = 1$
$g' = f$ changes from increasing to decreasing at $x = 1$.

(c) $x = -1,\ 1,\ 3$

(d) h is decreasing on $[0, 2]$
$h' = -f < 0$ when $f > 0$

16.

(a) $g(3) = \int_2^3 f(t)\,dt = \frac{1}{2}(4 + 2) = 3$
$g'(3) = f(3) = 2$
$g''(3) = f'(3) = \dfrac{0 - 4}{4 - 2} = -2$

(b) $$\frac{g(3) - g(0)}{3} = \frac{1}{3}\int_0^3 f(t)\,dt = \frac{1}{3}\left(\frac{1}{2}(2)(4) + \frac{1}{2}(4 + 2)\right) = \frac{7}{3}$$

(c) There are two values of c.

We need $\frac{7}{3} = g'(c) = f(c)$

The graph of f intersects the line $y = \frac{7}{3}$ at two places between 0 and 3.

(d) $x = 2$ and $x = 5$
because $g' = f$ changes from increasing to decreasing at $x = 2$, and from decreasing to increasing at $x = 5$.

17.	18.
(a) $f'(x) = 3\sqrt{4+(3x)^2}$ $g'(x) = f'(\sin x) \cdot \cos x$ $= 3\sqrt{4+(3\sin x)^2} \cdot \cos x$	(a) $f'(x) = \frac{1}{2}\left(25-x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25-x^2}}, \quad -5 < x < 5$
(b) $g(\pi) = 0,\ g'(\pi) = -6$ Tangent line: $y = -6(x-\pi)$	(b) $f'(-3) = \frac{3}{\sqrt{25-9}} = \frac{3}{4}$ $f(-3) = \sqrt{25-9} = 4$ An equation for the tangent line is $y = 4 + \frac{3}{4}(x+3)$.
(c) For $0 < x < \pi$, $g'(x) = 0$ only at $x = \frac{\pi}{2}$. $g(0) = g(\pi) = 0$ $g\left(\frac{\pi}{2}\right) = \int_0^3 \sqrt{4+t^2}\,dt > 0$ The maximum value of g on $[0, \pi]$ is $\int_0^3 \sqrt{4+t^2}\,dt$.	(c) $\lim_{x\to -3^-} g(x) = \lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \sqrt{25-x^2} = 4$ $\lim_{x\to -3^+} g(x) = \lim_{x\to -3^+} (x+7) = 4$ Therefore, $\lim_{x\to -3} g(x) = 4$. $g(-3) = f(-3) = 4$ So, $\lim_{x\to -3} g(x) = g(-3)$. Therefore, g is continuous at $x = -3$.
	(d) Let $u = 25 - x^2 \Rightarrow du = -2x\,dx$ $\int_0^5 x\sqrt{25-x^2}\,dx = -\frac{1}{2}\int_{25}^0 \sqrt{u}\,du$ $= \left[-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]_{u=25}^{u=0}$ $= -\frac{1}{3}(0-125) = \frac{125}{3}$

19.

(a) $\dfrac{(0+7)}{2}\cdot 8+\dfrac{(7+8)}{2}\cdot 6+\dfrac{(8+2)}{2}\cdot 8+\dfrac{(2+0)}{2}\cdot 2$

$= 115 \text{ ft}^2$

(b) $\dfrac{1}{120}\displaystyle\int_0^{120} 115v(t)\,dt$

$= 1807.169 \text{ or } 1807.170 \text{ ft}^3/\text{min}$

(c) $\displaystyle\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230 \text{ or } 122.231 \text{ ft}^2$

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is

$\dfrac{1}{20}\displaystyle\int_{40}^{60} C\cdot v(t)\,dt = 2181.912 \text{ or } 2181.913 \text{ ft}^3/\text{min}.$

Yes, water must be diverted since the average volumetri for this 20-minute period exceeds 2100 ft^3/min.

20.

(a) Midpoint Riemann sum is

$10\cdot[v(5)+v(15)+v(25)+v(35)]$

$= 10\cdot[9.2+7.0+2.4+4.3] = 229$

The integral gives the total distance in miles that t plane flies during the 40 minutes.

(b) By the Mean Value Theorem, $v'(t)=0$ somewhe the interval $(0, 15)$ and somewhere in the interva $(25, 30)$. Therefore the acceleration will equal 0 least two values of t.

(c) $f'(23) = -0.407$ or -0.408 miles per minute^2

(d) Average velocity $= \dfrac{1}{40}\displaystyle\int_0^{40} f(t)\,dt$

$= 5.916$ miles per minute

21.

(a) $\displaystyle\int_{30}^{60} |v(t)|\,dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\displaystyle\int_{30}^{60} |v(t)|\,dt$:

$A = \frac{1}{2}(14+10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185$ ft

(b) $\displaystyle\int_0^{30} a(t)\,dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$\displaystyle\int_0^{30} a(t)\,dt = \int_0^{30} v'(t)\,dt = v(30) - v(0)$

$= -14 - (-20) = 6$ ft/sec

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

Units of ft in (a) and ft/sec in (b)

22.

(a) Difference quotient; e.g.

$W'(12) \approx \dfrac{W(15)-W(12)}{15-12} = -\dfrac{1}{3}$ °C/day or

$W'(12) \approx \dfrac{W(12)-W(9)}{12-9} = -\dfrac{2}{3}$ °C/day or

$W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = -\dfrac{1}{2}$ °C/day

(b) $\dfrac{3}{2}(20+2(31)+2(28)+2(24)+2(22)+21) = 376.5$

Average temperature $\approx \dfrac{1}{15}(376.5) = 25.1$ °C

(c) $P'(12) = 10e^{-t/3} - \dfrac{10}{3}te^{-t/3}\Big|_{t=12}$

$= -30e^{-4} = -0.549$ °C/day

This means that the temperature is decreasing at the rate of 0.549 °C/day when $t = 12$ days.

(d) $\dfrac{1}{15}\displaystyle\int_0^{15}\left(20+10te^{-t/3}\right)dt = 25.757$ °C