Honors Calculus
Final Exam Review

1. Evaluate the integral: $\int (ax + b)\,dx$

(a) $\frac{ab}{2}x^2 + C$ (b) $a + C$ (c) $\frac{a}{2}x^2 + bx + C$

(d) $\frac{a}{2}x^2 + bx$ (e) None of these

2. Evaluate the integral: $\int \frac{3 + 4x^{3/2}}{\sqrt{x}}\,dx$.

(a) $\frac{3}{2}\sqrt{x} + 2x^2 + C$ (b) $-\frac{3}{2}x^{-3/2} + 4 + C$ (c) $\frac{3}{2}x^{-3/2} + 2x^2 + C$

(d) $6\sqrt{x} + 2x^2 + C$ (e) None of these

3. Evaluate the integral: $\int 3\csc^2 x\,dx$.

(a) $\frac{1}{3}\csc^3 x + C$ (b) $6\csc^2 x \cot x + C$ (c) $-3\cot x + C$

(d) $-\frac{1}{3}\csc^3 x + C$ (e) None of these

4. Evaluate the integral: $\int \frac{\sec^3\theta \tan\theta}{1 + \tan^2\theta}\,d\theta$.

(a) $\frac{1}{4}\sec^4\theta + C$ (b) $\frac{1}{2}\sec^2\theta + C$ (c) $\frac{1}{4}\sec^2\theta\tan^2\theta + C$

(d) $\sec\theta + C$ (e) None of these

5. Find the particular solution of the equation $f'(x) = 4x^{-1/2}$ that satisfies the condition $f(1) = 12$.

(a) $8\sqrt{x} + C$ (b) $2\sqrt{x} + 10$ (c) $\frac{1}{2}\sqrt{4x} + 11$

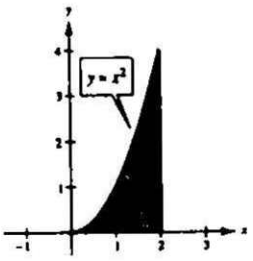
(d) $8\sqrt{x} + 4$ (e) None of these

6. Find $y = f(x)$ if $f''(x) = x^2$, $f'(0) = 7$ and $f(0) = 2$.

(a) $x^2 + 9$ (b) $\frac{1}{12}x^4 + 7x + 2$ (c) $x^2 + 7x + 2$

(d) $x^4 + 84x + 24$ (e) None of these

7. Which of the following definite integrals represents the area of the shaded region?



(a) $\int_0^4 x^2\,dx$ (b) $\int_0^2 x^2\,dx$

(c) $\int_1^2 x^2\,dx$ (d) $\int_0^2 x^2$

(e) None of these

8. Choose the correct statement given that $\int_0^9 f(x)\,dx = 5$ and $\int_3^9 f(x)\,dx = -1$.

(a) $\int_3^0 f(x)\,dx = 6$ (b) $\int_9^3 f(x)\,dx = -1$ (c) $\int_0^3 f(x)\,dx = 4$

(d) $\int_0^3 f(x)\,dx = 6$ (e) None of these

9. Use the Fundamental Theorem of Calculus to evaluate: $\int_0^2 |x - 1|\,dx$.

(a) 0 (b) 1 (c) $\frac{1}{2}$

(d) 2 (e) None of these

10. Find the average value of $f(x) = 2x^2 + 3$ on the interval $[0, 2]$.

(a) $\frac{22}{3}$ (b) $\frac{17}{3}$ (c) 4

(d) 27 (e) None of these

11. Find the area of the region bounded by $y = (x - 1)^2 + 1$, the x-axis, $x = -1$ and $x = 2$.

(a) 6 (b) 7 (c) -6

(d) $\frac{9}{2}$ (e) None of these

12. Use the general power rule to evaluate the integral: $\int x\sqrt{9 - 5x^2}\,dx$.

(a) $-\frac{1}{10}(9 - 5x^2)^{3/2} + C$ (b) $-\frac{1}{15}(9 - 5x^2)^{3/2} + C$ (c) $\frac{2}{3}(9 - 5x^2)^{3/2} + C$

(d) $-\frac{4}{15}(9 - 5x^2)^{3/2} + C$ (e) None of these

13. Evaluate the integral: $\int \frac{1}{\sqrt{2x + 1}}\,dx$.

(a) $\sqrt{x^2 + x} + C$ (b) $\frac{1}{\sqrt{x^2 + x}} + C$ (c) $\sqrt{2x + 1} + C$

(d) $\frac{1}{2}\sqrt{2x + 1} + C$ (e) None of these

14. Evaluate the integral: $\int \sin\frac{x}{2}\,dx$.

(a) $\cos\frac{x}{2} + C$ (b) $-2\cos\frac{x}{2} + C$ (c) $\sin\frac{x^2}{4} + C$

(d) $2\sin^2\frac{x}{2} + C$ (e) None of these

15. Evaluate the integral: $\int x\sec^2 x^2\,dx$.

(a) $\frac{1}{6}x^3\sec^3 x^2 + C$ (b) $\frac{1}{2}\tan x^2 + C$ (c) $\frac{1}{2}x^2\tan x^2 + C$

(d) $\tan x^3 + C$ (e) None of these

16. Find the derivative: $f(x) = \ln\frac{x(x^2+2)}{\sqrt{x^3-7}}$.

(a) $\frac{x^2+2}{x} + \frac{2x^2}{x^2+2} + \frac{3x^2}{2(x^3-7)}$ (b) $\frac{1}{x} + \frac{2x}{x^2+2} - \frac{3x^2}{2(x^3-7)}$

(c) $\frac{x^2+2}{x} + \frac{2x^2}{x^2+2} - \frac{3x^2}{2(x^3-7)}$ (d) $\frac{1}{x} + \frac{2x}{x^2+2} + \frac{3x^2}{2(x^3-7)}$

(e) None of these

17. Evaluate the integral: $\int_e^{4e}\frac{1}{x}\,dx$.

(a) $\ln 3e$ (b) $\ln 4$ (c) $-\frac{3}{4e}$

(d) $\frac{15}{16e^2}$ (e) None of these

18. Evaluate the definite integral: $\int_1^{\sqrt{e}}\frac{-7x}{x^2}\,dx$.

(a) -7 (b) $-\frac{7}{4}$ (c) $-\frac{7}{2}$

(d) $-\frac{21}{2}$ (e) None of these

19. Evaluate the integral: $\int\frac{x+2}{x+1}\,dx$.

(a) $\frac{x^2+4x}{x^2+2x} + C$ (b) $2x + C$ (c) $x + C$

(d) $x + \ln|x+1| + C$ (e) None of these

20. Solve the differential equation: $\frac{ds}{dt} = \frac{\sec t\tan t}{\sec t + 5}$.

(a) $s = \ln|\sec t + 5| + C$ (b) $s = \frac{1}{5}\ln|\sec t| + C$ (c) $s = 2\sec^3 t - \sec t + C$

(d) $s = \frac{1}{5}\tan t + C$ (e) None of these

21. Differentiate: $f(x) = \ln(e^{-x^2})$.

(a) e^{x^2} (b) $-2xe^{2x^2}$ (c) $-2x$

(d) $-2xe^{-x^2}$ (e) None of these

22. Find $f'(x)$ for $f(x) = \sqrt{4 + e^{2x}}$.

(a) $\dfrac{e^{2x}}{\sqrt{4 + e^{2x}}}$ (b) $\dfrac{1}{2\sqrt{2e^{2x}}}$ (c) $\dfrac{xe^{2x-1}}{\sqrt{4 + e^{2x}}}$

(d) e^x (e) None of these

23. Find $\dfrac{dy}{dx}$ if $xe^y + 1 = xy$.

(a) 0 (b) $\dfrac{y - e^y}{xe^y - x}$ (c) $\dfrac{y}{e^y - x}$

(d) $\dfrac{e^y}{xe^y - 1}$ (e) None of these

24. Find the slope of the tangent line to the graph of $y = (\ln x)e^x$ at the point where $x = 2$.

(a) $\frac{1}{2}e^2$ (b) $e^2\left(\ln 2 + \frac{1}{2}\right)$ (c) e

(d) $e(2 \ln 2 + 1)$ (e) None of these

25. Find $\dfrac{dy}{dx}$ if $y = \dfrac{x^3}{3^x}$.

(a) $\dfrac{x}{3^{x-2}}$ (b) $\dfrac{3x^2}{3^x(\ln 3)}$ (c) $\dfrac{x^2(9 - x^2)}{3^{x+1}}$

(d) $\dfrac{x^2[3 - x(\ln 3)]}{3^x}$ (e) None of these

26. Solve the differential equation: $2y' = y$.

(a) $y = Ce^{x/2}$ (b) $2y = \dfrac{y^2}{2} + C$ (c) $y = e^{2x} + C$

(d) $y = e^{x/2} + C$ (e) None of these

27. The number of fruit flies increases according to the law of exponential growth. If initially there are 10 fruit flies and after 6 hours there are 24, find the number of fruit flies after t hours.

(a) $y = 10e^{\ln(12/5)t/6}$ (b) $y = 10e^{\ln(12/5)t}$ (c) $y = 10e^{-\ln(12/5)t/6}$

(d) $y = 10e^{(\ln 12)t/6}$ (e) None of these

28. Find the general solution to the first order differential equation: $2x(y+1) - yy' = 0$.

(a) $2x^2 - y^2 \ln|y+1| = C$ (b) $x^2 = y + \frac{1}{(y+1)^2} + C$ (c) $\ln|y+1| + x^2 - y = C$

(d) $x^2(y+1)^2 - y^2 = C$ (e) None of these

29. Find the general solution of the differential equation: $xy' + 2y = 0$.

(a) $y = -2\ln|x| + C$ (b) $x^2y + 2y^2 = C$ (c) $y + 2x = C$

(d) $y = \frac{C}{x^2}$ (e) None of these

30. Differentiate: $f(x) = \arcsin\sqrt{1-4x^2}$.

(a) $\frac{2x}{|x|\sqrt{1-4x^2}}$ (b) $-\frac{2x}{|x|\sqrt{1-4x^2}}$ (c) $-\frac{2}{1-4x^2}$

(d) $\frac{2}{1-4x^2}$ (e) None of these

31. Find $\frac{dy}{dx}$ for $y = \arctan\frac{x}{2}$.

(a) $\frac{4}{4+x^2}$ (b) $\frac{4}{1+x^2}$ (c) $\frac{1}{\sqrt{4-x^2}}$

(d) $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ (e) $\frac{2}{4+x^2}$

32. Evaluate the integral: $\int \frac{1}{2x\sqrt{4x^2-1}}\,dx$.

(a) $\operatorname{arcsec}|2x| + C$ (b) $\frac{1}{2}\arcsin|2x| + C$ (c) $\frac{1}{8}\sqrt{4x^2-1} + C$

(d) $\frac{1}{2}\operatorname{arcsec}|2x| + C$ (e) None of these

33. Evaluate the integral: $\int \frac{x+3}{x^2+9}\,dx$.

(a) $\ln|x-3| + C$ (b) $\frac{1}{3}\arctan\frac{x}{3} + C$ (c) $\frac{1}{2}\ln(x^2+9) + \arctan\frac{x}{3} + C$

(d) $\ln(x^2+9) + \frac{1}{3}\arctan\frac{x}{3} + C$ (e) None of these

34. Evaluate the integral: $\int \frac{x}{81+x^4}\,dx$.

(a) $\frac{1}{2}\arcsin\frac{x^2}{9} + C$ (b) $\frac{1}{18}\arctan\frac{x^2}{9} + C$ (c) $\frac{1}{18}\operatorname{arcsec}\frac{x^2}{9} + C$

(d) $\frac{1}{9}\arctan\frac{x^2}{9} + C$ (e) None of these

35. Find the area of the region bounded by the graphs of $y = -x^2 + 2x + 3$ and $y = 3$.

(a) $\frac{4}{3}$ (b) $\frac{9}{2}$ (c) $\frac{22}{3}$

(d) $-\frac{4}{3}$ (e) None of these

36. Find the area of the region bounded by the graphs of $f(x) = 6x - x^2$ and $g(x) = x^2 - 2x$.

(a) 32 (b) $\frac{20}{3}$ (c) $\frac{64}{3}$

(d) 128 (e) None of these

37. Find the area of the region bounded by the graphs of $f(x) = 5x - x^2$ and $g(x) = 3x^2 + x$.

(a) $\frac{2}{3}$ (b) 6 (c) $\frac{1}{6}$

(d) $\frac{13}{6}$ (e) None of these

38. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$, $y = 1$, and $x = 2$ about the x-axis.

(a) $\frac{127}{7}\pi$ (b) $\frac{120}{7}\pi$ (c) $\frac{240}{7}\pi$

(d) $\frac{1013}{10}\pi$ (e) None of these

39. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$, $x = 2$, and $y = 1$ about the y-axis.

(a) $\frac{93}{5}\pi$ (b) $\frac{120}{7}\pi$ (c) $\frac{47}{5}\pi$

(d) $\frac{62}{5}\pi$ (e) None of these

40. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 3$, $y = 3$, $x = 0$, and $x = 2$ about the line $y = 3$.

(a) $\frac{112}{5}\pi$ (b) $\frac{32}{5}\pi$ (c) $\frac{56}{5}\pi$ (d) $\frac{64}{5}\pi$ (e) None of these

41. The rate of change of y with respect to x is inversely proportional to the square root of y.

a. Write a differential equation for the given statement.

b. Solve the differential equation in part **a**.

42. A radioactive element has a half-life of 50 days. What percentage of the original sample is left after 60 days?

43. Consider $F(x) = \int_1^{x^2} (t^3 + \sqrt{t})\, dt$. Find $F'(x)$.

44. Use the Trapezoidal Rule, with $n = 4$, to approximate $\int_2^3 \frac{1}{(x-1)^2}\, dx$.

45. Use the Midpoint Rule, with $n = 4$, to approximate the area of the region bounded by $f(x) = x^3 - 6x^2 + 9x$ and the x-axis between $x = 0$ and $x = 3$.