Rates of change and Rectilinear Motion

- Application of the Derivative: slope, slope of tangent line

- Rate of Change(definition): (change in y) ÷ change in x $\frac{\Delta y}{\Delta x}$

Examples:

- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________

When you are asked to find the rate of change, you are finding the slope.

Find the rate of change of $f(t) = (t^2 + 1)^3$ at $t = 1$

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions

1. position: $y = x(t)$

feet

Notation: $x(t), s(t), \ldots$

Application(s): find height at $t = 2$ sec, max height

2. Velocity:

feet/sec

Notation: $x'(t), s'(t), v(t)$

Application(s): velocity at $t = 1$

velocity at max height

3. Acceleration:

feet/sec^2

Notation: $x''(t), s''(t), v'(t), a(t)$

Application(s): find acceleration at $t = 2$

P
V
A
J

Ex 3: At time t=0 seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

a) When does the diver hit the water?

height of water = 0

$s(t) = 0$

$0 = -16t^2 + 16t + 32$

$0 = -16(t^2 - t - 2)$

$0 = -16(t-2)\cancel{(t+1)}$

$\boxed{t = 2\text{sec}}$

b) What is the diver's velocity at impact?

$s'(t) = -32t + 16$

$s'(2) = -64 + 16$

$= -48$ ft/sec

- Average Velocity vs. Instantaneous Velocity

$[a, b]$

- Average Velocity: (change in y) ÷ (change in x) $\frac{\Delta y}{\Delta x}$

  slope between 2 points

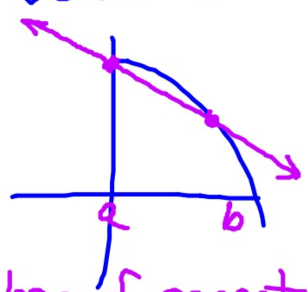
  Formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{s(b) - s(a)}{b - a}$

- Instantaneous Velocity: velocity at a given time

  Formula: $v(a) = \_\_\_$

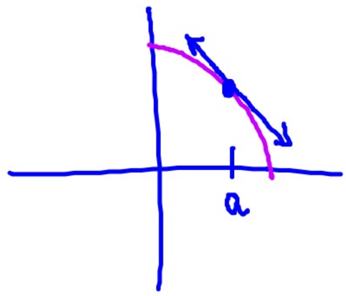
  $v(b) = \_\_\_$

Average velocity



Slope of secant

Instantaneous Velocity



- Speed: $|v(t)|$
- Rest: $v(t) = 0$
- Left and Right Motion:
  - Left: $v(t) < 0$ (negative)
  - Right: $v(t) > 0$ (positive)
  - Changes Direction: when $v(t)$ changes signs

Ex 4: A billiard ball is dropped from a height of 100 ft, its height *s* at time *t* is given by the position function $s(t) = -16t^2 + 100$, where *s* is measured in feet and *t* is measured in seconds.

a) Find the average velocity over time interval [1, 2].

$(1, s(1))$ $(2, s(2))$

$(1, 84)$ $(2, 36)$

$\dfrac{s(2)-s(1)}{2-1} = \dfrac{36-84}{2-1}$

$= -48$ ft/sec

b) Find the Instantaneous velocity at the endpoints of the interval.

$s'(1) = -32$ ft/sec

$s'(t) = -32t$

$s'(2) = -64$ ft/sec

c) Find the speed at the endpoints of the interval.

$|s'(1)| = 32$ ft/sec

$|s'(2)| = 64$ ft/sec

Ex 5: A particle starts at time t=0 and moves along the x-axis so that its position at any time $\boxed{t \geq 0}$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity of the particle at any time $t \geq 0$. Simplify.

$$x'(t) = \underbrace{(t-1)^3 \cdot 2} + \underbrace{(2t-3) \cdot 3(t-1)^2 \cdot 1}$$

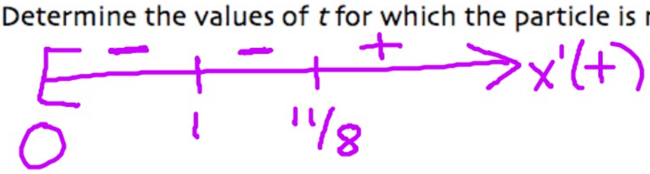
$$= (t-1)^2\big(2(t-1) + 3(2t-3)\big) = (t-1)^2(8t-11)$$

b) Determine the values of t for which the particle is at rest.

$$0 = (t-1)^2(8t-11)$$

$$t = 1,\ 11/8$$

c) Determine the values of t for which the particle is moving to the left. JYA.



Moving to the left on (0, 1) U (1, 11/8) because x'(t) < 0 on these intervals

d)
Moving to the right (11/8, inf) because x'(t) >0 on this interval

e) t = 11/8 sec because x'(t) changes signs at this time