## Solving Differential Equations: General and Particular solutions

general (Answer includes "c")

① $\frac{dy}{dx} = x\sqrt{x^2+1}$

$\int dy = \int x\sqrt{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$y = \int x\sqrt{u}\,\frac{du}{2x}$

$y = \frac{1}{2}\int \sqrt{u}\,du$

$y = \frac{1}{2}\int u^{1/2}\,du$

$y = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + c$

$y = \frac{1}{3}(x^2+1)^{3/2} + c$

(2) $\frac{dy}{dx} = x + 7$

$\int dy = \int (x+7)\,dx$

$y = \frac{x^2}{2} + 7x + C$

Separation of variables

③ $\frac{dy}{dx} = y + 6$

$\frac{dy}{dx} = \frac{y+6}{1}$

$\int \frac{dy}{y+6} = \int dx$

$u = y + 6$

$du = 1\,dy$

$\int \frac{du}{u} = x + C$

$\ln|u| = x + C$

$x^{2+3}$

$x^2 \cdot x^3$

$e^{\ln x} = x$

$e^{\ln|y+6|} = e^{x+C}$

$|y+6| = e^x \cdot e^C$ → $C$

$y + 6 = e^x \cdot C$

$y = Ce^x - 6$

(4) Particular solution : $y = f(t)$

$$\frac{dy}{dt} = \frac{1}{2}y \qquad (0, 4)$$

$$\int \frac{dy}{y} = \int \frac{1}{2}\,dt$$

$$\ln|y| = \frac{1}{2}t + C$$

$$\ln 4 = 0 + C$$

$$\ln 4 = C$$

$$e^{\ln|y|} = e^{\frac{1}{2}t + \ln 4}$$

$$y = e^{\frac{1}{2}t} \cdot e^{\ln 4} \to 4$$

$$y = 4e^{\frac{1}{2}t}$$

(5) $\frac{dy}{dx} = (4-y)$ $(0,1)$

$e^{\ln x} = x$

$\int \frac{dy}{4-y} = \int dx$

$-\ln|4-y| = x + C$

$-\ln 3 = C$

$-\ln|4-y| = x - \ln 3$

$\ln|4-y| = -x + \ln 3$

$e^{\quad} \qquad e^{\quad}$

$4-y = e^{-x} \cdot e^{\ln 3} \to 3$

$4-y = 3e^{-x}$

$y = 4 - 3e^{-x}$