Area : (top - bottom) or (right - left)

$y = -x^2 + 2$

$y = x$

Top - Bottom

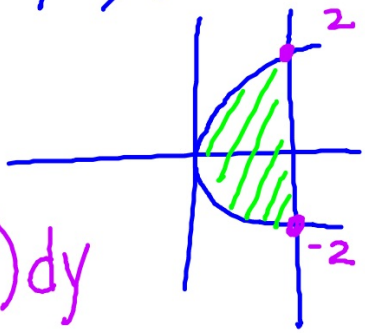
$\int_{-2}^{1} (-x^2 + 2 - x)\,dx$

$-x^2 + 2 = x$

$x^2 + x - 2 = 0$

$(x+2)(x-1)$

$x = y^2, \; x = 4$



$\int_{-2}^{2} (4 - y^2)\,dy$

Volume by cross-section

$\int_a^b \text{Area}\,dx$ or $\int_a^b \text{Area}\,dy$

a top-bottom = S a S = right-left

types of area

squares: $\text{Area} = S^2$

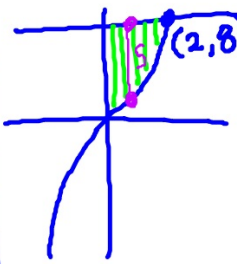
equil Δs: $\text{Area} = \frac{\sqrt{3}}{4} S^2$

semicircles: $\text{Area} = \frac{\pi}{8} S^2$

rectangles: $\text{Area} = C \cdot S$

$y = x^3$ $y = 8$ $x = 0$

$\perp$ x-axis, equil Δs.



$S = 8 - x^3$

$$\int_0^2 \frac{\sqrt{3}}{4} S^2\,dx$$

$$\frac{\sqrt{3}}{4}\int_0^2 (8-x^3)^2\,dx$$

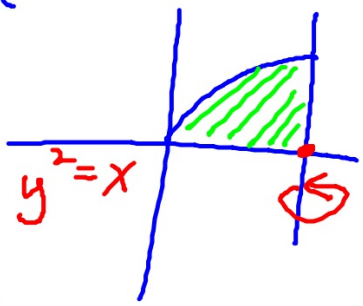
Volume by revolution

Disks: $(\text{function}-\text{axis})^2$

$y=\sqrt{x}$

$x=5$

$y=0$



rotate: $x=5$

$$\pi\int_0^{\sqrt{5}}\left(y^2-5\right)^2\,dy$$

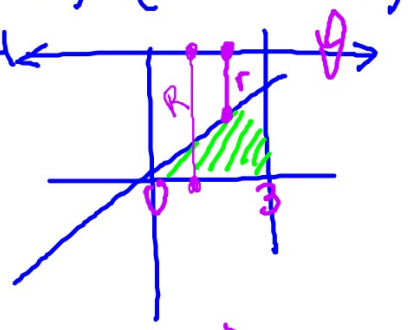
Washers: R^2-r^2

$(\text{outer fun.}-\text{axis})^2-(\text{inner fun.}-\text{axis})^2$

$y=x$

$x=3$

$y=0$

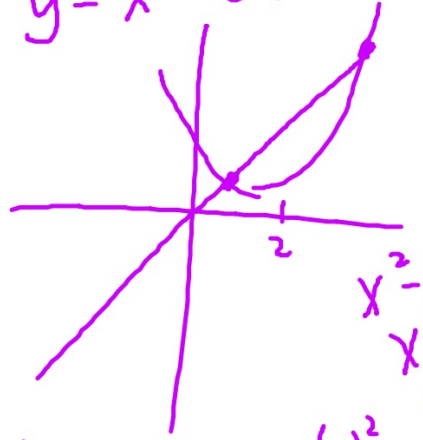


rotate: $y=4$

$$\pi\int_0^3\left((0-4)^2-(x-4)^2\right)dx$$

21.) $y = x$

$y = x^2 - 3x + 3$



$x^2 - 3x + 3 = x$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1)$

$y' = 2x - 3$

$0 = 2x - 3$

$1.5 = x$

$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 3$

$= \frac{9}{4} - \frac{9}{2} + 3$

$\int_1^3 \left(x - \left(x^2 - 3x + 3\right)\right) dx$

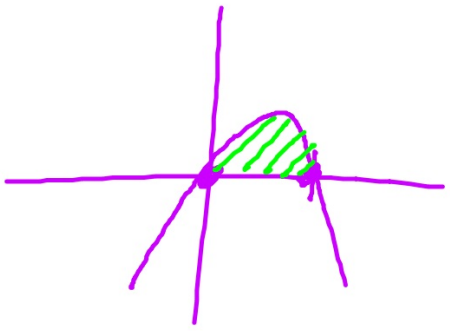
$\int_1^3 \left(-x^2 + 4x - 3\right) dx$

$-\frac{x^3}{3} + 2x^2 - 3x \Big|_1^3$

$\left(-9 + 18 - 9\right) - \left(-\frac{1}{3} + 2 - 3\right)$

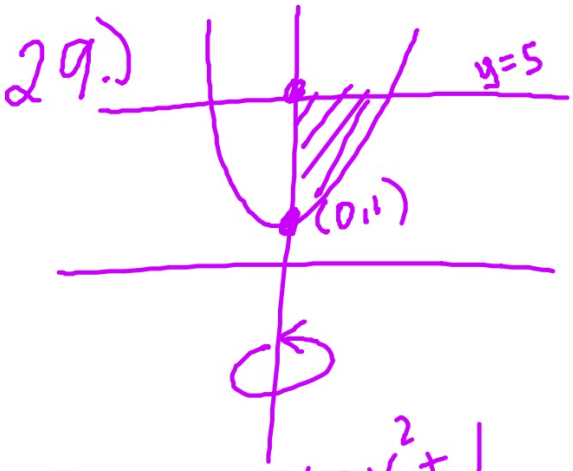
$\frac{1}{3} - 2 + 3 = \frac{4}{3}$

1. $y = x(1-x) = x - x^2$



$$\int_0^1 (x - x^2 - 0)\,dx$$

29.)



disk "y"

$y = x^2 + 1$

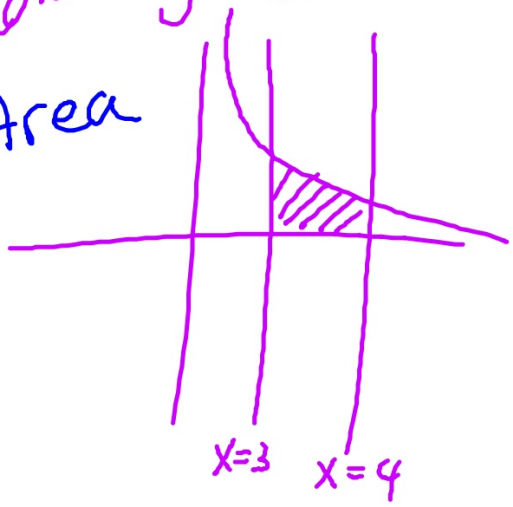
$\sqrt{y-1} = x$

$$\pi \int_1^5 (\text{funct.} - \text{axis})^2 \, dy$$

$$\pi \int_1^5 (\sqrt{y-1} - 0)^2 \, dy$$

6.) $y = \frac{1}{x-1}$

Area



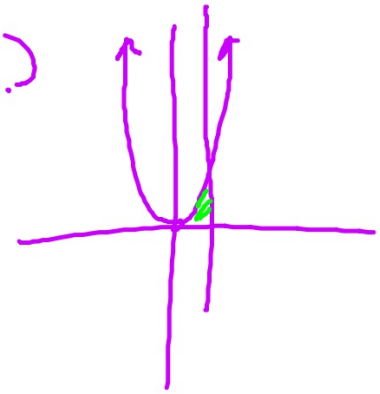
$$\int_3^4 \left(\frac{1}{x-1} - 0\right) dx$$

$$\ln|x-1| \Big|_3^4$$

$$\ln 3 - \ln 2$$

$$\ln \frac{3}{2}$$

25.)



$\perp$ x-axis

square

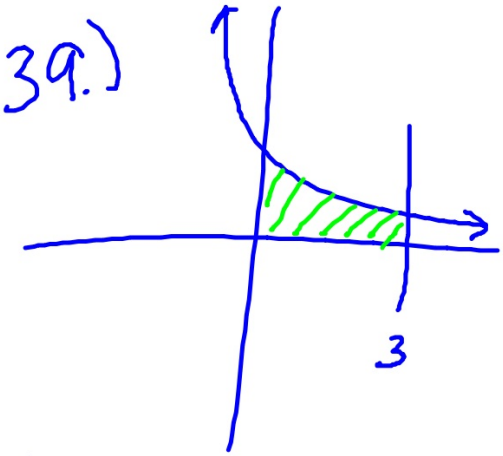
$S = 4x^2 - 0$

$$\int_0^1 S^2\,dx = \int_0^1 (4x^2)^2\,dx$$

$$= \int_0^1 16x^4\,dx = \frac{16x^5}{5}\Big|_0^1$$

$$= \frac{16}{5}$$

39.)



⊥ x-axis squares

$s = e^{-x} - 0$

$s^2 = e^{-2x}$

$$\int_0^3 s^2\,dx = \int_0^3 e^{-2x}\,dx$$

$u = -2x$

$du = -2dx$

$\frac{du}{-2} = dx$

$$-\frac{1}{2}\int e^u\,du$$

$$= -\frac{1}{2}e^{-2x}\Big|_0^3 = \frac{-1}{2}\left(e^{-6} - 1\right)$$

$$-\frac{1}{2}e^{-6} + \frac{1}{2}$$

(A)

30.) $y = \sec x$

$x = 0, y = 0, x = \frac{\pi}{3}$

"disk"

"x"

$$\pi \int_0^{\pi/3} (\sec x - 0)^2 \, dx$$

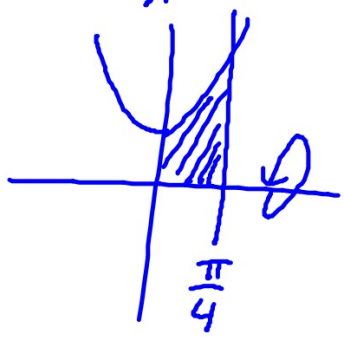
$$\pi \cdot \tan x \Big|_0^{\pi/3}$$

$$\pi \left(\sqrt{3}\right)$$

$$\int \tan x \, dx = -\ln|\cos x| + C$$

35.) $y = \sec x$

$x = \frac{\pi}{4}$



disk

"x"

$$\pi \int_0^{\pi/4} (\sec x - 0)^2 \, dx$$

$$\pi \cdot \tan x \Big|_0^{\pi/4}$$

$$\pi(1) = \pi$$