9.) $y y' = 4\sin x$

$$\frac{y\,dy}{dx} = \frac{4\sin x}{1}$$

$$\int y\,dy = \int 4\sin x\,dx$$

$$2\left(\frac{1}{2}y^2 = -4\cos x + C\right)$$

$$y^2 = -8\cos x + C \leftarrow$$

$$y = \pm\sqrt{-8\cos x + C}$$

11.) $\sqrt{1-4x^2}\, y' = x$

$$\sqrt{1-4x^2}\,\frac{dy}{dx} = x$$

$$\int dy = \int \frac{x\,dx}{\sqrt{1-4x^2}}$$

$u = 1-4x^2$

$du = -8x\,dx$

$-\frac{1}{8}du = dx$

$$y = -\frac{1}{8}\int u^{-1/2}\,du$$

$$y = -\frac{1}{8}\cdot\frac{u^{1/2}}{1/2} + C$$

$$y = -\frac{1}{4}\sqrt{1-4x^2} + C$$

Omit #4 in Part 2 on the review

Also look at #6 and #7 on the 6.2 WS

5.) $\frac{dw}{dt} = kt$

$\int dw = \int kt\,dt$

$W = \frac{kt^2}{2} + C$

or

$W = kt^2 + C$

6.) $\int \frac{dy}{\cancel{dx}} = \int \frac{k}{x}\,dx$

$y = k\ln|x| + C$

7.) $\frac{dy}{dx} = kx - x$

$\int dy = \int (kx - x)\, dx$

$y = \frac{kx^2}{2} - \frac{x^2}{2} + C$

$y = kx^2 - \frac{x^2}{2} + C$

6.) $\frac{dy}{dx} = (y+k)$

$\int \frac{dy}{y+k} = \int dx$

$\ln|y+k| = x + c$

$e \qquad e$

$y + k = e^x \cdot e^c \rightarrow c$

$y = Ce^x - k$

or

$y = Ce^x + k$

10) $(2, 3)$ $(4, \frac{3}{4})$

$y = ab^x$

$3 = ab^2$

$\frac{3}{b^2} = a$

$\frac{3}{\frac{1}{4}} = a$

$12 = a$

$\frac{3}{4} = ab^4$

$\frac{3}{4} = \frac{3}{b^2} b^4$

$\frac{3}{4} = 3b^2$

$\frac{1}{4} = b^2$

$\frac{1}{2} = b$

$y = 12\left(\frac{1}{2}\right)^x$

5.) $\frac{dy}{dx} = y + ky$

$\frac{dy}{dx} = y(1+k)$

$\int \frac{dy}{y} = \int (1+k)\,dx$

$e^{\ln|y|} = e^{x+kx+c}$

$y = e^{x} \cdot e^{kx} \cdot e^{c} \rightarrow c$

$y = Ce^{x+kx}$

$y = Ce^{x(1+k)}$

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20.) $\frac{dy}{dt} = \frac{3}{4}y \quad (0,10)$

$$\int \frac{dy}{y} = \int \frac{3}{4}\,dt$$

$$\ln|y| = \frac{3}{4}t + C$$

$$\ln 10 = C$$

$$e^{\ln|y|} = e^{\frac{3}{4}t + \ln 10}$$

$$y = e^{\frac{3}{4}t} \cdot e^{\ln 10}$$

$$y = 10e^{\frac{3}{4}t}$$

8) $$\frac{dy}{dx} = ky(x-1)$$

$$\int \frac{dy}{y} = \int k(x-1)\,dx$$

$$\ln|y| = k\left(\frac{x^2}{2} - x\right) + C$$

$$\ln|y| = kx^2 - kx + C$$

$e \qquad e$

$$y = e^{kx^2} \cdot e^{-kx} \cdot e^{C} \rightarrow c$$

$$y = Ce^{kx^2 - kx}$$

or

$$y = Ce^{kx^2 + kx}$$

$(2,3)$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + C$$

$$\ln 3 = \ln 2 + C$$

$$\ln 3 - \ln 2 = C$$

$$\ln \frac{3}{2} = C$$

$$e^{\ln|y|} = e^{\ln|x| + \ln \frac{3}{2}}$$

$$y = e^{\ln x} \cdot e^{\ln \frac{3}{2}}$$

$$y = \frac{3}{2} x$$

9.) $\dfrac{dy}{dx} = \dfrac{x(y-2)}{x^2+4}$ $\quad (1, 5)$

$$\int \frac{dy}{y-2} = \int \frac{x}{x^2+4}\,dx$$

$u = x^2 + 4$

$du = 2x\,dx$

$\dfrac{du}{2x} = dx$

$$\ln|y-2| = \int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$\ln|y-2| = \frac{1}{2}\ln|x^2+4| + C$$

$\ln 3 = \frac{1}{2}\ln 5 + C$

$\ln 3 = \ln\sqrt{5} + C$

$\ln 3 - \ln\sqrt{5} = C$

$\ln \dfrac{3}{\sqrt{5}} = C$

$$\ln|y-2| = \frac{1}{2}\ln|x^2+4| + \ln\frac{3}{\sqrt{5}}$$

$$e^{\ln|y-2|} = e^{\ln\sqrt{x^2+4} + \ln\frac{3}{\sqrt{5}}}$$

$$e^{\ln x} = x$$

$$y-2 = e^{\ln\sqrt{x^2+4}} \cdot e^{\ln\frac{3}{\sqrt{5}}}$$

$$y-2 = \sqrt{x^2+4} \cdot \frac{3}{\sqrt{5}}$$

$$y = \frac{3\sqrt{x^2+4}}{\sqrt{5}} + 2$$

#6 $\left(0, \frac{1}{2}\right)$ $(5, 5)$ $(10, \_\_\_)$

6.2 WS

$e^{\ln x} = x$

rate of change of y is proportional to y

$\left(0, \frac{1}{2}\right)$ $(5, 5)$

$\frac{dy}{dx} = ky$

$\frac{1}{2} = Ce^{0}$

$\frac{1}{2} = C$

$5 = \frac{1}{2}e^{5k}$

$\ln 10 = \ln e^{5k}$

$\ln 10 = 5k$

$\frac{1}{5}\ln 10 = k$

$y = Ce^{kx}$

$= \frac{1}{2}e^{\left(\frac{1}{5}\ln 10\right)(10)} = \frac{1}{2}e^{2\ln 10} = \frac{1}{2}e^{\ln 100} = \frac{1}{2}(100)$

$= 50$

15.) $yy' - 2e^x = 0$ $\quad (0,3)$

$$y\frac{dy}{dx} = 2e^x$$

$$\int y\,dy = \int 2e^x\,dx$$

$$\frac{1}{2}y^2 = 2e^x + C$$

$$\frac{9}{2} = 2 + C$$

$$\frac{5}{2} = C$$

$$2\left(\frac{1}{2}y^2 = 2e^x + \frac{5}{2}\right)$$

$$y^2 = 4e^x + 5$$

$$y = \pm\sqrt{4e^x + 5}$$

$$\boxed{y = \sqrt{4e^x + 5}}$$

13.) $y\ln x - xy' = 0$

$$y\ln x = x\frac{dy}{dx}$$

$u = \ln x$
$du = \frac{1}{x}dx$
$x\,du = dx$

$$\int \frac{\ln x}{x}dx = \int \frac{dy}{y}$$

$$\int \frac{u}{\cancel{x}}\cancel{x}\,du = \ln|y|$$

$$\frac{u^2}{2} = \ln|y| + C$$

$$\frac{(\ln x)^2}{2} + C = \ln|y|$$

$$e \qquad e$$

$$e^{\frac{1}{2}(\ln x)^2} \cdot e^{C} = y$$

$$Ce^{\frac{1}{2}(\ln x)^2} = y$$

$$(\ln x)^2 \neq \ln x^2$$