79.) $\int x \cdot 5^{-x^2} dx$

$u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$\int x \cdot 5^u \cdot \frac{du}{-2x}$

$-\frac{1}{2}\int 5^u du = -\frac{1}{2} \cdot \frac{1}{\ln 5} \cdot 5^u + C$

$-\frac{1}{2\ln 5} \cdot 5^{-x^2} + C$

Anti-derivatives:

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{1}{\ln a} \cdot a^u + C$$

Also, all trig anti-derivatives

degree num. ≥ degree den

· divide

Solve differential equations

Find the area enclosed in a region

Find the average value

Evaluate

① $\int_0^2 \frac{x^2}{x^3+1}\,dx$

② $\int x e^{-x^2}\,dx$

③ $\int \frac{x^2-x+1}{x-1}\,dx$

Evaluate

① $\int_0^2 \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\ln|x^3+1|\Big|_0^2 = \frac{1}{3}\ln 9 = \ln\sqrt[3]{9}$

② $\int x e^{-x^2}\,dx = -\frac{1}{2}e^{-x^2} + C$

③ $\int \frac{x^2-x+1}{x-1}\,dx = \int \left(x + \frac{1}{x-1}\right)dx = \frac{1}{2}x^2 + \ln|x-1| + C$

$$\begin{array}{r|rrr} 1 & 1 & -1 & 1 \\ & & 1 & 0 \\ \hline & 1 & 0 & 1 \end{array}$$

4) $\int (x-1)3^{x^2-2x}\,dx$

5) $\int_0^1 \frac{e^{4x}}{e^{4x}+1}\,dx$

6) $\int \cot 4x\,dx$

④ $\int (x-1)3^{x^2-2x}dx = \frac{1}{2}\cdot\frac{1}{\ln 3}\cdot 3^{x^2-2x} + C$

⑤ $\int_0^1 \frac{e^{4x}}{e^{4x}+1}dx = \frac{1}{4}\ln|e^{4x}+1|\Big|_0^1 = \frac{1}{4}\left(\ln|e^4+1| - \ln 2\right)$

$u = e^{4x}+1$

$du = 4e^{4x}dx$

⑥ $\int \cot 4x\,dx = \frac{1}{4}\ln|\sin 4x| + C$

7. $\int \frac{3x^2 - x + 7}{x}\,dx$

8. $\int \csc 2x\,dx$

⑦ $\int \frac{3x^2 - x + 7}{x}\,dx = \int \left(3x - 1 + \frac{7}{x}\right)dx = \frac{3x^2}{2} - x + 7\ln|x| + C$

⑧ $\int \csc 2x\,dx = -\frac{1}{2}\ln|\csc 2x + \cot 2x| + C$

Solve the differential equation.

9) $\frac{dy}{dx} = 1 + \frac{1}{x}$ $(1,4)$

$y = x + \ln|x| + C$

$4 = 1 + \ln|1| + C$

$3 = C$

$\boxed{y = x + \ln|x| + 3}$

10) $\frac{dy}{dx} = \tan\frac{x}{2}$ $(\frac{\pi}{2}, 0)$

$y = -2\ln|\cos\frac{x}{2}| + C$

$0 = -2\ln\frac{\sqrt{2}}{2} + C$

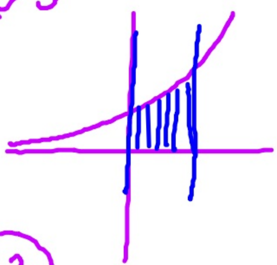
$2\ln\frac{\sqrt{2}}{2} = C$

$\ln\frac{2}{4} = C$

$\boxed{y = -2\ln|\cos\frac{x}{2}| + \ln\frac{1}{2}}$

Sketch. Find the area of the region enclosed by

11 $y = e^{4x}$, $x = 0$, $x = 1$, $y = 0$



$$\int_0^1 e^{4x}\,dx = \frac{1}{4}e^{4x}\Big|_0^1 = \frac{1}{4}\left(e^4 - 1\right)$$

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Find the average value of $y = \frac{\sin x}{1+\cos x}$ on $\left[0, 3\pi/4\right]$

$$\frac{1}{\frac{3\pi}{4} - 0}\int_0^{3\pi/4} \frac{\sin x}{1+\cos x}\,dx = -\frac{4}{3\pi}\ln\left|1+\cos x\right|\Big|_0^{3\pi/4} = \frac{-4}{3\pi}\left(\ln\left|1-\frac{\sqrt{2}}{2}\right| - \ln 2\right)$$