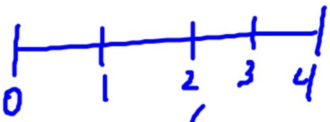
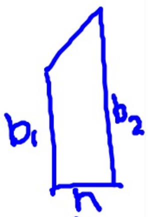
$f(x) = \frac{1}{2}x^3 + 1$ on $[0,4]$ $\underline{n=4}$



$w = \frac{4-0}{4} = 1$

Right: $1\left(f(4)+f(3)+f(2)+f(1)\right)$



Midpoint: $1\left(f\left(\frac{1}{2}\right)+f\left(1\frac{1}{2}\right)+f\left(2\frac{1}{2}\right)+f\left(3\frac{1}{2}\right)\right)$

Trapezoid: $\frac{1}{2}(1)\left[f(0)+f(1)+f(1)+f(2)+f(2)+f(3)+f(3)+f(4)\right]$

x	3	5	9	12	17
g(x)	2	4	8	6	5

$n = 4$

left: $2(g(3)) + 4g(5) + 3(g(9)) + 5g(12)$

$2 \cdot 2 + 4 \cdot 4 + 3 \cdot 8 + 5 \cdot 6$

trapezoid: $\frac{1}{2}\left[2(2+4) + 4(4+8) + 3(8+6) + 5(6+5)\right]$

① $\int (x^3 - 2x + 1)\,dx$

$\frac{x^4}{4} - x^2 + x + C$

② $\int \frac{\sqrt{x} + 1}{x^2}\,dx$

$\int (x^{-3/2} + x^{-2})\,dx$

$-2x^{-1/2} + -1x^{-1} + C$

③ $\int \csc x \cot x\,dx$

$-\csc x + C$

④ $\int (1 + \tan^2 x)\,dx = \int \sec^2 x\,dx$

$= \tan x + C$

⑤ $\int_1^{16} \frac{1}{\sqrt{x}}\,dx = \int_1^{16} x^{-1/2}\,dx$

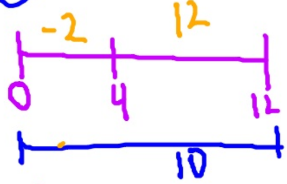
$= 2x^{1/2}\Big]_1^{16} = 2(4-1)$

6

⑥ $\int (4\sin x - 2\cos x)\,dx$

$-4\cos x - 2\sin x + C$

$$\int_0^4 f(x)\,dx = -2 \qquad \int_0^{12} f(x)\,dx = 10$$



① $\int_4^{12} f(x)\,dx = 12$

② $\int_{12}^{4} f(x)\,dx = -12$

③ $\int_0^4 2f(x)\,dx$

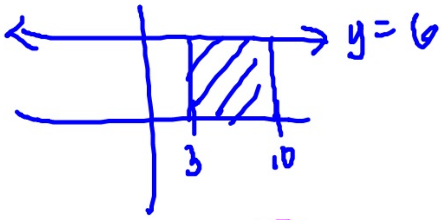
-4

④ $\int_4^{12} (3f(x) - 5)\,dx = \int_4^{12} 3f(x)\,dx - \int_4^{12} 5\,dx$

$36 - 40$

-4

$$\int_3^{10} g(x)\,dx = 11$$



⑤ $$\int_3^{10} \left(6 - 4g(x)\right)dx = \int_3^{10} 6\,dx - 4\int_3^{10} g(x)\,dx$$

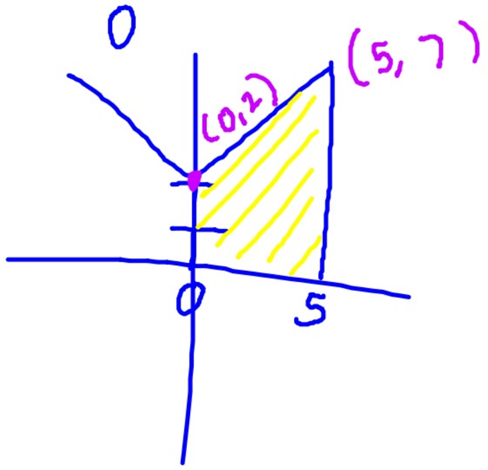
$$6x\Big]_3^{10}$$

$$60 - 18$$

$$42 - 4(11)$$

$$-2$$

$$\int_0^5 (|x| + 2)\,dx$$

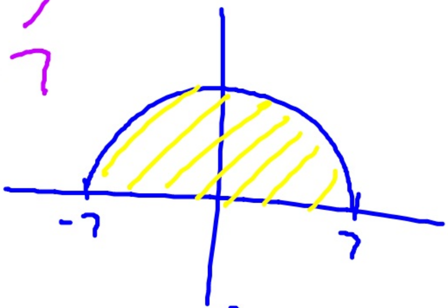


$$\frac{1}{2} h (b_1 + b_2)$$

$$\frac{1}{2}(5)(2+7)$$

$$\frac{45}{2}$$

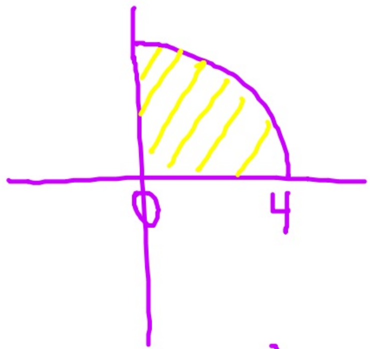
① $\int_{-7}^{7} \sqrt{49 - x^2}\,dx$



$\frac{1}{2}\pi(7)^2$

$\frac{49\pi}{2}$

② $\int_{0}^{4} \sqrt{16 - x^2}\,dx$



$\frac{1}{4}\pi r^2$

4π

Average value $y = \cos 2x \quad \left[0, \frac{\pi}{12}\right]$

$$\frac{1}{\frac{\pi}{12} - 0} \int_0^{\pi/12} \cos 2x \, dx$$

$$\frac{1}{2} \cdot \frac{12}{\pi} \int_0^{\pi/12} \cos 2x \, dx = \frac{6}{\pi} \int \cos u \, du = \frac{6}{\pi} \sin u$$

$$\frac{6}{\pi} \sin 2x \Big]_0^{\pi/12} = \frac{6}{\pi}\left(\frac{1}{2} - 0\right)$$

$$\frac{3}{\pi}$$

$u = 2x$

$\frac{du}{2} = \frac{\cancel{2} dx}{\cancel{2}}$

$$\int \frac{dy}{\cancel{dx}} = \int x(1-x^2)^5\,dx$$

$$y = \int x(1-x^2)^5\,dx = -\frac{1}{2}\int u^5\,du$$

$u = 1-x^2$

$\frac{du}{-2} = \frac{\cancel{-2}x\,dx}{\cancel{-2}}$

$$y = -\frac{1}{2}\cdot\frac{u^6}{6} + C$$

$$y = \frac{-(1-x^2)^6}{12} + C$$

general solution

$$F(x) = \int_{3}^{x^2} \sin t^3 \, dt$$

$$F'(x) = \sin x^6 \cdot 2x$$

$$g(x) = \int_{3}^{x^3} \sqrt{1+t}\,dt$$

$$g'(x) = \sqrt{1+x^3} \cdot 3x^2$$

$$g'(2) = \sqrt{9} \cdot 12$$

$$= 36$$

61.) $f''(x) = 2$ $\quad f'(2) = 5$ $\quad f(2) = 10$

Solve the D.E.

$f'(x) = \int 2\,dx$

$f'(x) = 2x + C$

$5 = 2(2) + C$

$1 = C$

$f'(x) = 2x + 1$

$f(x) = \int (2x+1)\,dx$

$f(x) = x^2 + x + C$

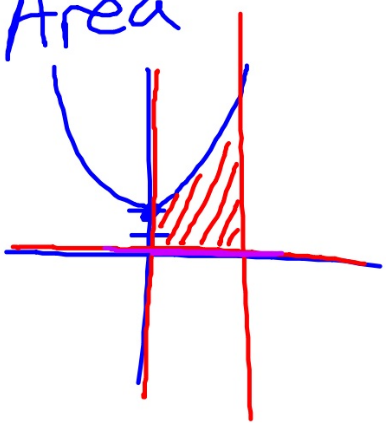
$10 = 4 + 2 + C$

$4 = C$

$f(x) = x^2 + x + 4$

39.) $y = 5x^2 + 2$ $x=0$ $x=2$ $y=0$

Area



$$\int_0^2 (5x^2+2)\,dx$$

$= \square$

91.) $F(x) = \int_0^{x^3} \sin t^2 \, dt$

$F'(x) = \sin x^6 \cdot \underline{\underline{3x^2}}$

95.) $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$

Av. value

$$\frac{1}{\frac{2\pi}{3}-\frac{\pi}{2}}\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$$

$$\frac{6}{\pi}\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$$

$$u=\frac{x}{2}=\frac{1}{2}x$$

$$\frac{du}{1/2}=\frac{\frac{1}{2}dx}{\frac{1}{2}}$$

$$\frac{6}{\pi}\cdot 2\int \sec^2 u\,du$$

$$\frac{12}{\pi}\tan u$$

$$\frac{12}{\pi}\tan\left(\frac{x}{2}\right)\Big]_{\pi/2}^{2\pi/3}$$

$$\frac{12}{\pi}\left[\tan\frac{\pi}{3}-\tan\frac{\pi}{4}\right]$$

$$\frac{12}{\pi}\left(\sqrt{3}-1\right)$$

73.) $\displaystyle\int \frac{\cos\theta}{\sqrt{1-\sin\theta}}\,d\theta = -\int \frac{du}{\sqrt{u}} = -\int u^{-1/2}\,du$

$u = 1-\sin\theta$

$du = 0 - \cos\theta\,d\theta$

$\displaystyle = -\frac{u^{1/2}}{1/2} + C$

$-2\sqrt{1-\sin\theta} + C$

$-\cancel{2\cdot\tfrac{1}{2}}(1-\sin\theta)^{-1/2}(+\cos\theta)$

$\displaystyle\frac{\cos\theta}{\sqrt{1-\sin\theta}}$

① $\int \frac{x^2+x^3}{x^5}\,dx$

$\int x^{-3}+x^{-2}\,dx$

② $\int \sin 2x\,dx$

u-sub

③ $\int \sec x \tan x\,dx = \sec x + C$

trig

④ $\int x^5\,dx$

power

⑤ $\int \frac{1}{\sqrt{x}}\,dx$ $\int x^{-1/2}dx$

rewrite

⑥ $\int \frac{x}{\sqrt{x^2+1}}\,dx$

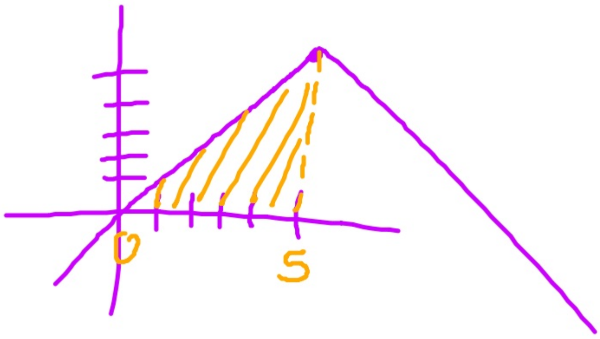
u-sub

power
re-write
trig
u-sub

37.) $\int_0^5 (5-|x-5|)\,dx$

$y = 5-|x-5|$

$y = -|x-5| + 5$



$\frac{1}{2}(5)(5)$

12.5

56.) $y = x + \cos x$

$$\int_0^{\pi/2} (x + \cos x)\,dx = \frac{x^2}{2} + \sin x \Big]_0^{\pi/2}$$