Implicit Differentiation
Related Rates
Motion on a Line
Absolute Extrema on a closed interval
Absolute vs relative extrema
Critical numbers
Rolle's Theorem
Mean Value Theorem

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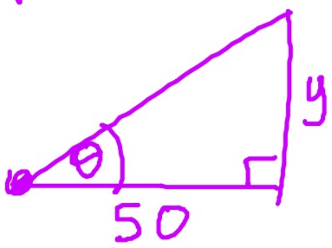
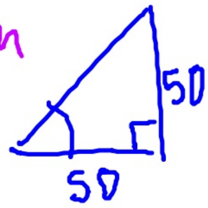


$$\frac{dy}{dt} = 4 \text{ m/sec}$$

$$\frac{d\theta}{dt} = ? \text{ when } y = 50\text{m}$$



$$\frac{d}{dt}\left(\tan\theta = \frac{y}{50}\right)$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{50}\frac{dy}{dt}$$

$$\left(\sqrt{2}\right)^2 \frac{d\theta}{dt} = \frac{4}{50}$$

$$\frac{d\theta}{dt} = \frac{4}{50}\cdot\frac{1}{2}$$

$$= \frac{1}{25} \text{ rad/sec}$$

2)

$$\frac{d\theta}{dt} = 0.2 \text{ rad/min}$$

$$\frac{dy}{dt} = ?$$

$$\tan\theta = \frac{y}{2}$$

$$\frac{d}{dt}\left(\tan\theta = \frac{1}{2}y\right)$$

$$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{2} \cdot \frac{dy}{dt}$$

$$\left(\sec\frac{\pi}{6}\right)^2 (.2) = \frac{1}{2}\frac{dy}{dt}$$

$$\frac{2}{1} \cdot \left(\frac{2}{\sqrt{3}}\right)^2 \left(\frac{1}{5}\right) = \frac{dy}{dt}$$

$$\frac{8}{15} \text{ miles/min} = \frac{dy}{dt}$$

1.) $\frac{dV}{dt} = -4 \text{ cm}^3/\text{min}$ $\quad \frac{dr}{dt} = ?$ when $V = 36\pi$?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-4 = 4\pi(3)^2 \frac{dr}{dt}$$

$V = \frac{4}{3}\pi r^3$

$36\cancel{\pi} = \frac{4}{3}\cancel{\pi} r^3$

$27 = r^3$

$3 = r$

11.) $f(x) = 2\sin x - x \qquad (0, 2\pi)$

$f'(x) = 2\cos x - 1$

$0 = 2\cos x - 1$

$\frac{1}{2} = \cos x$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

8.) $f(x) = (x-1)^2(x-3)$

$f'(x) = \underline{(x-1)^2(1)} + \underline{(x-3)\cdot 2(x-1)}$

$= (x-1)\left(x - 1 + 2(x-3)\right)$

$0 = (x-1)(3x-7)$

$x = 1, 7/3$

4.) $h(x) = 3\sqrt{x} - x$ $[0,9]$

$h'(x) = \frac{3}{2}x^{-1/2} - 1$

$= \frac{3}{2\sqrt{x}} - 1$

$0 = \frac{3-2\sqrt{x}}{2\sqrt{x}}$

$3 - 2\sqrt{x} = 0$

$3 = 2\sqrt{x}$

$\frac{3}{2} = \sqrt{x}$

$\frac{9}{4} = x$

x	y
0	0
9/4	9/4
9	0

$3 \cdot \sqrt{\frac{9}{4}} - \frac{9}{4}$

$\frac{9}{2} - \frac{9}{4}$

max $(9/4, 9/4)$

min $(0,0), (9,0)$

14. $f(x) = \frac{1}{x}$ $[1,4]$

MVT ?

cont $[1,4]$ ✓

diff. $(1,4)$ ✓

$f'(x) = \frac{-1}{x^2}$

$\frac{+1}{x^2} = \frac{+1}{4}$

$x^2 = 4$

$x = \pm 2$

$\boxed{x = 2}$

$(1, 1)\ (4, 1/4)$

$\frac{\frac{1}{4} - 1}{4 - 1} = \frac{4 \cdot \frac{1}{4} - 1 \cdot 4}{3 \cdot 4}$

$= \frac{1 - 4}{12} = \frac{-3}{12}$

$= \frac{-1}{4}$

13.) $y = x^{2/3}$ $[1, 8]$

cont. ✓

diff ✓

$y' = \frac{2}{3} x^{-1/3}$

$(1, 1)$

$(8, 4)$

$\frac{4-1}{8-1} = \frac{3}{7}$

$\cancel{\frac{2}{3}} x^{-1/3} = \frac{3}{7} \cdot \frac{3}{2}$

$x^{-1/3} = \frac{9}{14}$

$x^{+1/3} = \frac{14}{9}$

$x = \frac{14^3}{9^3}$

9.) $f(x) = x^{3/2} - 3x^{1/2}$ $\sqrt{x}(x-3)$

$f'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$

$= \frac{3}{2}x^{-1/2}\left(x - 1\right)$

$f'(x) = \frac{3(x-1)}{2\sqrt{x}}$

$x = 0, 1$

3.) $\frac{dr}{dt} = 0.4 \text{ ft/sec}$ $\qquad \frac{dA}{dt} = ?$ when $r = 5\text{ft}$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(5)\left(\frac{4}{10}\right)$$

$$= 4\pi \text{ ft}^2/\text{sec}$$

41.) $\frac{d}{dx}\left(\frac{x^2}{2}+\frac{y^2}{8}=1\right)$

tan. line

$(1, 2)$

$$\frac{2x}{2}\frac{dx}{dx}+\frac{2y}{8}\frac{dy}{dx}=0$$

$$\frac{x^2}{2}=\frac{1}{2}x^2$$

$$x+\frac{y}{4}\frac{dy}{dx}=0$$

$$y-2=-2(x-1)$$

$$\frac{y}{4}\cdot\frac{dy}{dx}=-x$$

$$\frac{dy}{dx}=\frac{-4x}{y}$$

Implicit

$$\frac{d}{dx}\left(3x^2 + 4\boxed{xy} - 6y^3 = 10\right)$$

$$6x\frac{\cancel{dx}}{\cancel{dx}} + 4\left(x\frac{dy}{dx} + y\cdot\frac{\cancel{dx}}{\cancel{dx}}\right) - 18y^2\frac{dy}{dx} = 0$$

$$6x + 4x\frac{dy}{dx} + 4y - 18y^2\frac{dy}{dx} = 0$$

$$3x + 2x\frac{dy}{dx} + 2y - 9y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3x-2y}{2x-9y^2} = \frac{3x+2y}{-2x+9y^2}$$

2a.) $(x^2+4)y = 8$

$$\frac{d}{dx}\left(x^2y + 4y = 8\right)$$

$$x^2 \cdot 1 \cdot \frac{dy}{dx} + y \cdot 2x \cancel{\frac{dx}{dx}} + 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-4}{8}$$

Average velocity

$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{9-1}{3-1} = \frac{8}{2} = 4$$

$$\boxed{\begin{array}{l} y = x^2 \\ \quad [1, 3] \end{array}}$$

$(1, 1)\ (3, 9)$

(e) $s(t) = 80t - 16t^2$

$0 = 16t(5 - t)$

$t = 5 \text{ sec.}$

$s'(t) = 80 - 32t$

$s'(5) = 80 - 32(5)$

$= -80 \text{ ft/sec}$

$$y = (x+2)^{1/3} + 8$$

$$y' = \frac{1}{3}(x+2)^{-2/3} \cdot 1 + 0$$

$$y' = \frac{1}{3(x+2)^{2/3}}$$

$$x = -2$$

10.) $f(x) = \cos^2 x - \sin x \qquad (0, 2\pi)$

$f'(x) = 2\cos x(-\sin x) - \cos x \qquad (\cos x)^2 - \sin x$

$= -2\sin x \cos x - \cos x$

$0 = -\cos x(2\sin x + 1)$

$\cos x = 0 \qquad \sin x = \frac{-1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$W(x) = \sin 2x \qquad [0, 2\pi]$

cont. ✓

diff ✓

$W(0) = W(2\pi)$

$0 = 0$ ✓

$$W'(x) = 2\cos 2x$$

$$2\cos 2x = 0$$

$$\cos 2x = 0$$

let $A = 2X$

$$\cos A = 0$$

$$2X = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

2π 2π

$$X = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$g(x) = \frac{x^2 - 9}{3x}$$

$[1, 4]$

$(1, -8/3)(4, \frac{7}{12})$

$$m = \frac{12 \cdot \frac{7}{12} + \frac{8}{3} \cdot 12}{3 \cdot 12}$$

$$= \frac{7 + 32}{36}$$

$$= \frac{39}{36} = \frac{13}{12}$$

$$g'(x) = \frac{3x(2x) - (x^2 - 9)3}{9x^2}$$

$$= \frac{3x^2 + 27}{9x^2}$$

$$= \frac{x^2 + 9}{3x^2}$$

$$\frac{x^2 + 9}{3x^2} = \frac{13}{12}$$

$$39x^2 = 12x^2 + 108$$

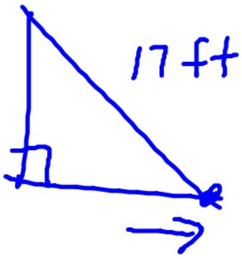
$$27x^2 = 108$$

$$x^2 = 4$$

$\cancel{x = \pm 2}$

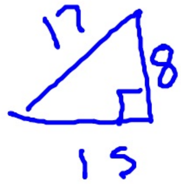
2

4)



$\frac{dx}{dt} = .5$ ft/sec

$\frac{dy}{dt} = \_\_$ when $y = 8$ ft



$$x^2 + y^2 = c^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$

$$15\left(\frac{1}{2}\right) + 8\left(\frac{dy}{dt}\right) = 0$$

$$8\frac{dy}{dt} = \frac{-15}{2}$$

$$\frac{dy}{dt} = \frac{-15}{16} \text{ ft/sec}$$

$$x^3 + y^3 = 6xy \qquad (3, 3)$$

$$3x^2 \frac{d\cancel{x}}{\cancel{dx}} + 3y^2 \frac{dy}{dx} = 6\left(x \frac{dy}{dx} + y \cdot \frac{d\cancel{x}}{\cancel{dx}}\right)$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + 6y$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

-1