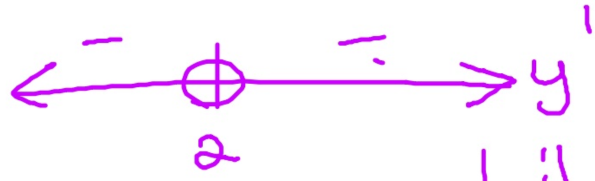
3.3: Relative Extrema/Intervals of Increasing/decreasing
(1st derivative test to find relative extrema)
Sketching f' and f'' given f

3.4: POI/Intervals of Concavity
2nd derivative test to find relative extrema

decr.
~~$(-\infty, \infty)$~~

3.6: Curve sketching given a function
Curve sketching given characteristics

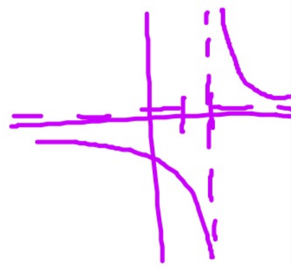
3.7: Optimization



$$y = \frac{1}{x-2}$$

$$y = (x-2)^{-1}$$

$$y' = -(x-2)^{-2} \cdot 1 = \frac{-1}{(x-2)^2}$$



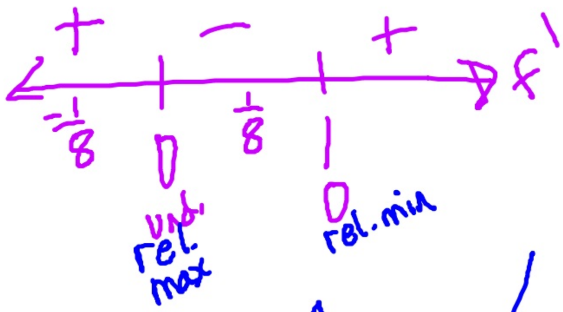
$f(x) = 2x - 3x^{2/3}$

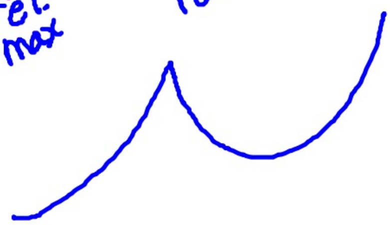
D: $(-\infty, \infty)$

$f'(x) = 2 - 2x^{-1/3}$

$= 2 - \dfrac{2}{x^{1/3}}$

$f'(x) = \dfrac{2x^{1/3} - 2}{x^{1/3}}$





slope 0

$2x^{1/3} - 2 = 0$

$x = 1$

slope undefind

$x = 0$

2c.)
max volume

Primary

$V = lwh$

$V = (24-2x)(24-2x)x$

$V = (24^2 - 96x + 4x^2)x$

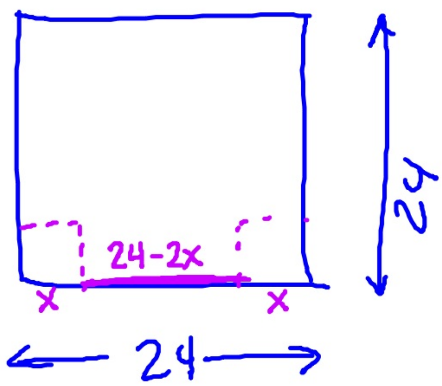
$V = 576x - 96x^2 + 4x^3$

$x = 4$

$+ \quad | \quad - \quad V'$

4

max value
$V = 1024 \text{ in}^3$



$V' = -192x + 12x^2 + 576$

$0 = 12x^2 - 192x + 576$

$0 = 12(x^2 - 16x + 48)$

$12(x-12)(x-4)$

$x = \cancel{12}, 4$

2(6.)

Primary

max area

$A = xy$

$A = x\left(\frac{6-x}{2}\right)$

$A = \frac{1}{2}(6x - x^2)$

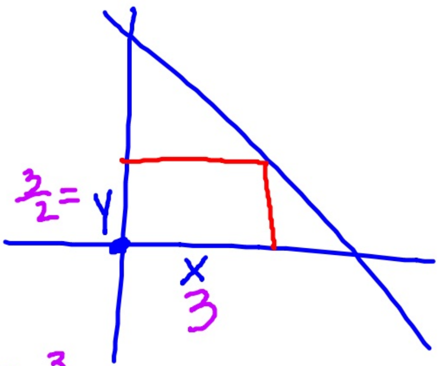
$A' = \frac{1}{2}(6 - 2x)$ A' ← + | − → (3)

$x = 3$

Secondary

$y = \frac{6-x}{2}$

$y = \frac{6-3}{2} = \frac{3}{2}$



$\boxed{3, \frac{3}{2}}$

p.223

#22: $50, \frac{200}{3}$

37.) $f(x) = (x+9)^2$

$f'(x) = 2(x+9)^1 \cdot 1 = 2x+18$

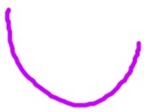
$0 = (x+9)$

$-9 = x$

$f''(x) = 2$

$f''(-9) > 0$

rel. min @ $(-9, 0)$



rel. extrema points!

25.) $h(x) = \sqrt{x}(x-3)$ $\boxed{x > 0}$

$$h(x) = x^{3/2} - 3x^{1/2}$$

$$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$$

$$= \frac{3x^{1/2}}{2} - \frac{3}{2x^{1/2}}$$

$$h'(x) = \frac{3x-3}{2x^{1/2}}$$

$x = 1$

relative min at (1,-2)
because h' changes
from neg. to pos.
at this point.

h is decreasing on (0,1) because
h'< 0 on this interval.

h is increasing on (1, inf) because
h' > 0 on this interval

22.) p. 224

Primary

max. area

$A = 2xy$

$A = 2x\left(\frac{400-4x}{3}\right)$

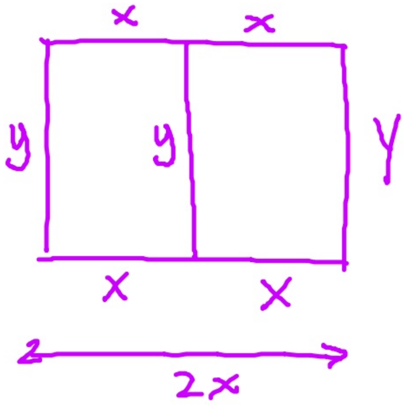
$A = \frac{2}{3}(400x - 4x^2)$

$A' = \frac{2}{3}(400 - 8x)$

Secondary

$400 = 4x + 3y$

$\frac{400-4x}{3} = y$



39.) $g(x) = 2x^2 - 2x^4$

$g'(x) = 4x - 8x^3$

$0 = 4x(1 - 2x^2)$

$x = 0, \pm\frac{1}{\sqrt{2}}$

$\frac{2}{2} - 2\left(\frac{1}{4}\right)$

$g''(x) = 4 - 24x^2$

$g''(0) > 0$ ccu rel. min $(0,0)$

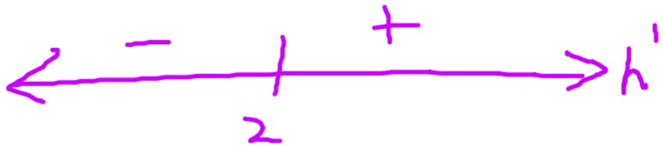
$g''\left(\frac{1}{\sqrt{2}}\right) < 0$ CCD rel. max $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

$g''\left(-\frac{1}{\sqrt{2}}\right) < 0$ CCD rel. max $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

29.) $h(t) = \frac{1}{4}t^4 - 8t$

$h'(t) = t^3 - 8$

$t = 2$



relative min at (2, -12) because
h' changes from negative
to positive at this point.

35.) $f(x) = x + \cos x$ $\quad [0, 2\pi]$

$f'(x) = 1 - \sin x$

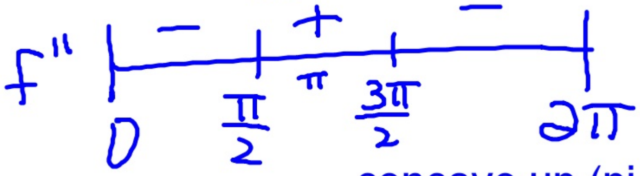
$f''(x) = -\cos x$

$0 = -\cos x$

POI: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

POI at these points because
f'' changes signs at these
points.



concave up (pi/2, 3pi/2) because
f''> 0 on this interval

concave down (0, pi/2) and (3pi,2, 2pi)
becaue f'' < 0 on these intervals.

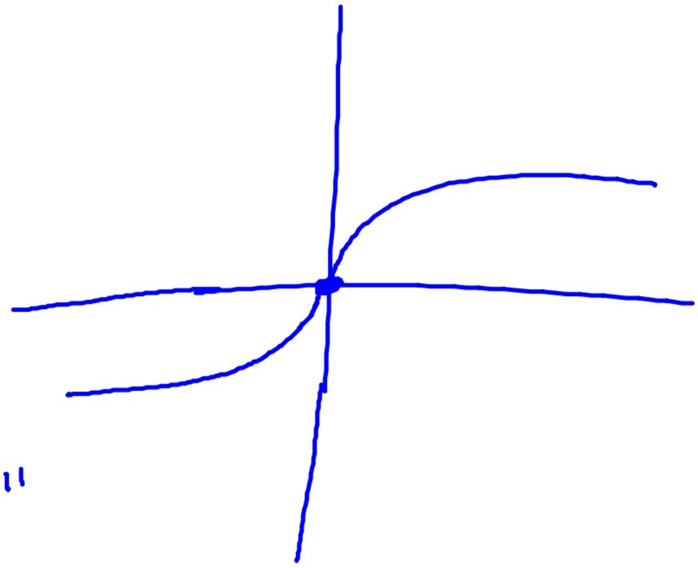
Part 4

$f(x) > 0 \quad x > 0$

$f(x) < 0 \quad x < 0$

$$\xleftarrow{\quad + \quad} \underset{0}{|} \xrightarrow{\quad + \quad} f'$$

$$\xleftarrow{\quad + \quad} \underset{0}{|} \xrightarrow{\quad - \quad} f''$$



Part 3

3.) $P = xy$ $\qquad x + 5y = 80$

$x = 80 - 5y$

$P = y(80 - 5y)$

$P = 80y - 5y^2$

$P' = 80 - 10y$

$8 = y$