3.) $f(x) = 4x^2 - 6x + 3$

$f(4) = 64 - 24 + 3$
$= 43$

<u>check</u>
$f'(x) = 8x - 6$
$f'(4) = 32 - 6$
$= 26$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(4) = \lim_{x \to 4} \frac{4x^2 - 6x + 3 - (43)}{x - 4}$$

$$= \lim_{x \to 4} \frac{4x^2 - 6x - 40}{x - 4} = \lim_{x \to 4} \frac{2(2x^2 - 3x - 20)}{x - 4}$$

$$= \lim_{x \to 4} \frac{2\cancel{(x-4)}(2x+5)}{\cancel{x-4}} = \lim_{x \to 4} 2(2x+5) = 26$$

9.) $f(t) = 5t^2 + 7t - \cancel{\frac{4}{t}}\ 4t^{-1}$

$f'(t) = 10t + 7 + 4t^{-2}$

$= 10t + 7 + \frac{4}{t^2}$

16) $f(x) = 5x\cot(x^2)$

$$f'(x) = 5x \cdot \left(-\csc^2(x^2) \cdot 2x\right) + \cot(x^2) \cdot 5$$

$$= -5\left(2x^2\csc^2(x^2) - \cot(x^2)\right)$$

17) $y = -2(4x+7)^{-1/2}$

$y' = 1(4x+7)^{-3/2} \cdot 4$

$y' = \frac{4}{(4x+7)^{3/2}}$

20.) $f(x) = (1-x)^{1/3}$ $\quad c = -7$

$f'(x) = \frac{1}{3}(1-x)^{-2/3}(-1)$

$f'(-7) = -\frac{1}{3}(8)^{-2/3}$

$= -\frac{1}{3} \cdot \frac{1}{4}$

$= \frac{-1}{12}$

22.) $f(x) = \dfrac{\cot x}{x}$

$$f'(x) = \frac{x(-\csc^2 x) - \cot x \cdot 1}{x^2}$$

$$f'\left(\tfrac{\pi}{4}\right) = \frac{-\frac{\pi}{4}(2) - 1 \cdot 1}{\frac{\pi^2}{16}}$$

$$= \frac{-\frac{\pi}{2} - 1}{\frac{\pi^2}{16}} \qquad 16 \quad \cdot 16 \quad \cdot 16 = \frac{-8\pi - 16}{\pi^2}$$

$$\boxed{y - \frac{4}{\pi} = \frac{-8\pi - 16}{\pi^2}\left(x - \frac{\pi}{4}\right)}$$

25.) $f(x) = \frac{x^2}{x} + \frac{1}{x} = x + x^{-1}$

$f'(x) = 1 - x^{-2}$ $(1, 2)$

$= 1 - \frac{1}{x^2}$ $(-1, -2)$

$0 = \frac{x^2 - 1}{x^2}$

$0 = x^2 - 1$

$x = \pm 1$

27.) $g(x) = (x^2+1)^5$

$g'(x) = 5(x^2+1)^4 \cdot 2x$

$g'(x) = 10x(x^2+1)^4$

$g''(x) = 10\left(x \cdot 4(x^2+1)^3(2x) + (x^2+1)^4 \cdot 1\right)$

$= 10\left((x^2+1)^3(8x^2 + x^2+1\right)$

$= 10(x^2+1)^3(9x^2+1)$

32.)

$$\lim_{x\to 1^-}(2-x) = \lim_{x\to 1^+}(x^2-2x+2) = f(1)$$

$$1 = 1$$

yes; $f(x)$ is cont.

$$f'(x) = \begin{cases} -1, & x<1 \\ 2x-2, & x>1 \end{cases}$$

$$\lim_{x\to 1^-}(-1) = \lim_{x\to 1^+}(2x-2) = f'(1)$$

$$-1 \neq 0$$

$f(x)$ is not diff at $x=1$

14.) $y = \dfrac{x-4}{x^2+1}$

$$y' = \frac{(x^2+1)\cdot 1 - (x-4)2x}{(x^2+1)^2}$$

$$y' = \frac{x^2+1-2x^2+8x}{(x^2+1)^2}$$

$$= \frac{-x^2+8x+1}{(x^2+1)^2}$$

15) $h(t) = \underset{(\sin 4x)^2}{\sin^2(4t)}$

$h'(t) =$

$2(\sin 4x)^1 \cdot \cos 4x \cdot 4$

$= 8\sin 4x\cos 4x$

16) $f(x) = 5x\cot(x^2)$

$f'(x) = 5x(-\csc^2(x^2)) \cdot 2x + \cot(x^2) \cdot 5$

$= -10x^2\csc^2(x^2) + 5\cot(x^2)$

$= -5(2x^2\csc^2(x^2) - \cot(x^2))$

21.) $g'(x) = \sec(5x)\tan(5x) \cdot 5$

$g'\left(\frac{\pi}{3}\right) = \sec\frac{5\pi}{3} \cdot \tan\frac{5\pi}{3} \cdot 5$

$= (2) \cdot (-\sqrt{3}) \cdot 5$

$= -10\sqrt{3}$

$\frac{d^2y}{dx^2}$

29.) $m(x) = f(g(x))$

$m'(x) = f'(g(x)) \cdot g'(x)$

$= f'(g(2))\, g'(2)$

$f'(3) \cdot (2) = 4$

32.) $f(x) = \begin{cases} 2 - x, & x \le 1 \\ x^2 - 2x + 2, & x > 1 \end{cases}$ $f'(x) = \begin{cases} -1, & x < 1 \\ 2x - 2, & x > 1 \end{cases}$

Continuity?

$$\lim_{x \to 1^-} (2 - x) = \lim_{x \to 1^+} (x^2 - 2x + 2) = f(1)$$

$$1 = 1$$

Diff?

$$\lim_{x \to 1^-} (-1) = \lim_{x \to 1^+} (2x - 2) = f'(1)$$

$$-1 \neq 0$$

not diff at $x = 1$

33)

$$\lim_{x\to 1^-}(a-bx^3)=\lim_{x\to 1^+}(b+4x)=f(1)$$

$$a-b=b+4$$

$$a=2b+4$$

$$f'(x)=\begin{cases}-3bx^2 & x\le 1\\ 4 & , x>1\end{cases}$$

$$\lim_{x\to 1^-}(-3bx^2)=\lim_{x\to 1^+}4=f'(1)$$

$$-3b=4$$

$$b=-4/3$$

5) $C(x) = g(f(x))$

$$C'(x) = g'(f(x)) \cdot f'(x)$$

$$= g'(f(1)) \cdot f'(1)$$

$$= g'(4) \cdot 5$$

$$= 2 \cdot 5$$

$$10$$

25.) $f(x) = \dfrac{x^2+1}{x} = x + \dfrac{1}{x} = x + x^{-1}$

$f'(x) = 1 - 1x^{-2}$ $\quad (1, \ 2)$

$= 1 - \dfrac{1}{x^2}$ $\quad (-1, \ -2)$

$0 = \dfrac{x^2-1}{x^2}$

$x^2 - 1 = 0$

$x = \pm 1$

22) $f(x) = \dfrac{\cot x}{x}$ $\left(\dfrac{\pi}{4}, \dfrac{4}{\pi}\right)$

$$f'(x) = \frac{x(-\csc^2 x) - \cot x \cdot 1}{x^2}$$

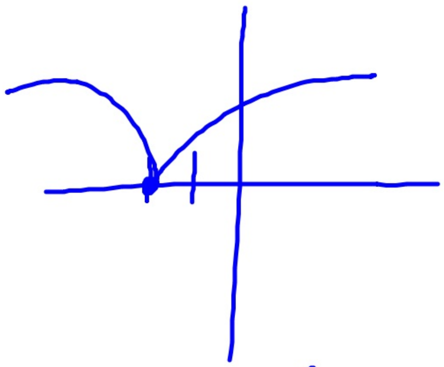
$$\boxed{y - \frac{4}{\pi} = \frac{-8\pi - 16}{\pi^2}\left(x - \frac{\pi}{2}\right)}$$

$$f'\left(\frac{\pi}{4}\right) = \frac{\frac{\pi}{4}\left(-\csc^2 \frac{\pi}{4}\right) - \cot \frac{\pi}{4}}{\frac{\pi^2}{16}}$$

$$= \frac{\frac{\pi}{4}(-2) - 1}{\frac{\pi^2}{16}} = \frac{16 \cdot \frac{-\pi}{2} - 1 \cdot 16}{\frac{\pi^2}{16} \cdot 16} = \frac{-8\pi - 16}{\pi^2}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

6.) $g(x) = (x+2)^{2/3}$



$(-\infty, -2) \cup (-2, \infty)$