

2) $f(x) = x^2 - x - 12$ $[-5, -1]$

$$f(-5) = 18$$

$$f(-1) = -10$$

~~A graph of the function is shown, showing a parabola opening upwards with a hole at (-3, 0). The x-axis is labeled from -5 to 1, and the y-axis is labeled from -12 to 2. The parabola passes through (-5, 18), (-4, 12), (-3, 0), (-2, 0), (-1, -10), and (0, -12).~~

Since $f(x)$ is continuous on $[-5, -1]$ and $f(-1) < 0 < f(-5)$, by IVT there must exist a value c such that

$$f(c) = 0 \quad 0 = x^2 - x - 12$$

$$0 = (x-4)(x+3)$$

$$x = 4, -3$$

$x = 4$ is circled in pink.

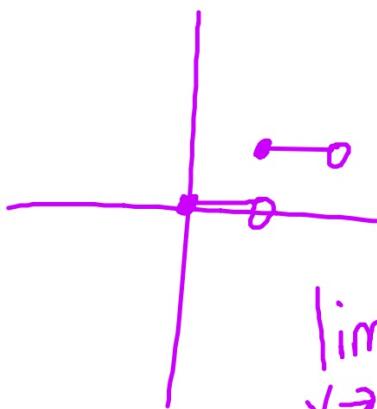
$$10.) \lim_{x \rightarrow 6^-} 3[x-3]$$

$$3[5.9-3]$$

$$3[2.9]$$

$$3.2$$

$$6$$



$$\lim_{x \rightarrow 1} [x] \\ \text{dne}$$

$$\lim_{x \rightarrow 1^-} [x] \neq \lim_{x \rightarrow 1^+} [x]$$

$$7.) \lim_{x \rightarrow 2} \frac{3x^2 - 4x - 4}{2x^2 - 8}$$

$$\lim_{x \rightarrow 2} \frac{(3x+2)(x-2)}{2(x+2)(x-2)}$$

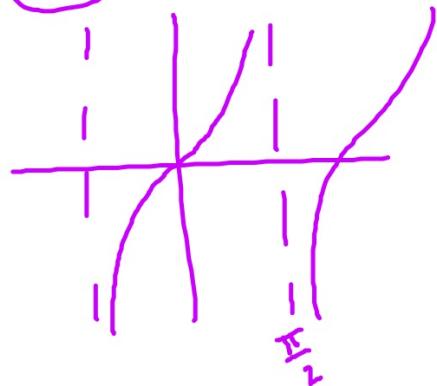
$$\frac{8}{8}$$

1

$$\lim \tan x$$

$$x \rightarrow \frac{\pi}{2}^-$$

(D) or $-\infty$



$$8.) \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} \cdot \frac{(\sqrt{x+1} + 1)}{(\sqrt{x+1} + 1)}$$

$$\lim_{x \rightarrow 0} \frac{x + x - x}{x(\sqrt{x+1} + 1)}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + 1}$$

$$\frac{1}{2}$$

$$19.) f(x) = \begin{cases} 3x+a, & x \leq -3 \\ ax^2+4, & x > -3 \end{cases}$$

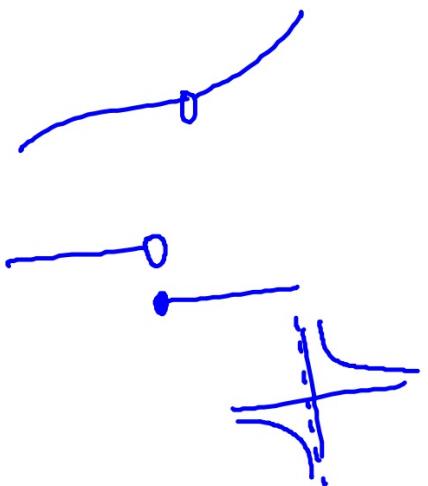
$$\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x) = f(-3)$$

$$\lim_{x \rightarrow -3^-} (3x+a) = \lim_{x \rightarrow -3^+} (ax^2+4)$$

$$-9+a = 9a+4$$

$$-13 = 8a$$

$$-13/8 = a$$



$$20.) f(x) = \begin{cases} 2x - 3ax^2, & x < 1 \\ bx - 4, & 1 \leq x < 2 \\ x^2 + a, & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

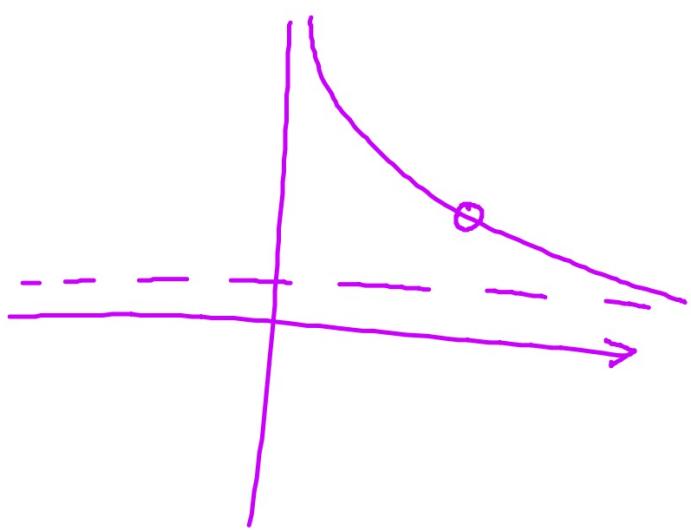
$$2 - 3a = b - 4$$

$$\begin{matrix} 3a + b = 6 \\ a - 2b = -8 \end{matrix}$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$$2b - 4 = 4 + a$$

$$a - 2b = -8$$



$$2) \lim_{x \rightarrow 0} \frac{\sin 4x}{\pi x}$$

$\frac{1}{\pi} \lim_{x \rightarrow 0} \frac{\sin 4x}{x} \cdot \frac{4}{4}$

$\frac{4}{\pi} \left[\lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \right]$

$\frac{4}{\pi} \cdot 1$
 $\frac{4}{\pi}$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$7.) \lim_{x \rightarrow 2} \frac{3x^2 - 4x - 4}{2x^2 - 8}$$

$$\lim_{x \rightarrow 2} \frac{(3x+2)(x-2)}{2(x+2)(x-2)}$$

$$\frac{8}{8}$$

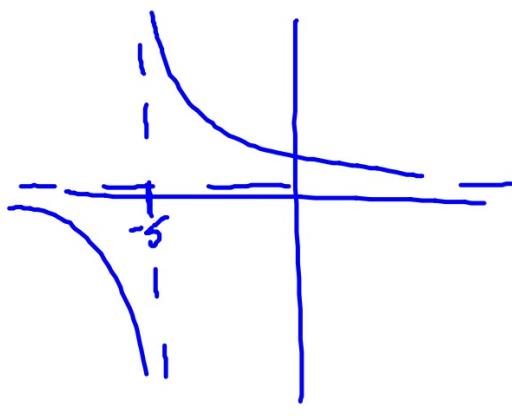
1

$$6.) \lim_{x \rightarrow -5} \frac{x-5}{x^2-25}$$

$$\lim_{x \rightarrow -5} \frac{x-5}{(x+5)(x-5)}$$

$$\lim_{x \rightarrow -5} \frac{1}{x+5}$$

∞ or $-\infty$ or dne



$$9.) \lim_{x \rightarrow 0} \frac{1}{2^x}$$

|

$$4.) \lim_{x \rightarrow 2} f(x)$$

$$f(x) = \begin{cases} 3x^2 - 7 & x \leq 2 \\ x^3 + 1 & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^-} \frac{1}{x-2}$$

$$\lim_{x \rightarrow 1^-} [x]$$

$$\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$$

$$\lim_{x \rightarrow 2^-} (3x^2 - 7) \neq \lim_{x \rightarrow 2^+} (x^3 + 1)$$

$$5 \neq 9$$

$$10.) \lim_{x \rightarrow 6^-} 3[x - 3]$$

$$16.) \lim_{x \rightarrow -\infty} \frac{7x^2 - 9}{4x + 3}$$

$$3[5.9 - 3]$$

$$3[2.9]$$

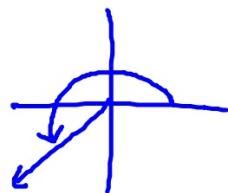
$$3 \cdot 2$$

$$6$$

Botn
 ∞ or $-\infty$
+
—

$$\text{II.) } \lim_{x \rightarrow 5} \csc \frac{\pi x}{4}$$

$$\csc \frac{5\pi}{4}$$



$$\sin \frac{\pi}{4} \\ \frac{\sqrt{2}}{2}$$

$$-\sqrt{2}$$

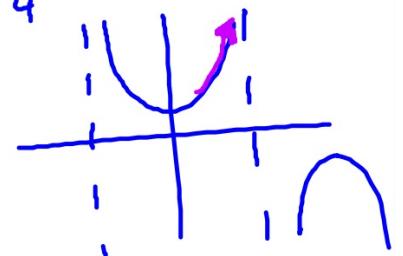
$$\lim_{x \rightarrow 2} \sec \frac{\pi x}{4}$$

$$\sec \frac{\pi}{2}$$

dne

$$\lim_{x \rightarrow 2^-} \sec \frac{\pi x}{4}$$

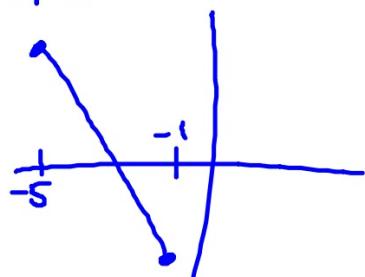
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$$21.) \quad f(x) = x^2 - x - 12 \quad [-5, -1] \\ f(c) = 0$$

$$f(-5) = 18$$

$$f(-1) = -10$$



Since $f(x)$ is continuous on $[-5, -1]$ and $f(-1) < 0 < f(-5)$, by IVT there exists a value c such that $f(c) = 0$

$$0 = x^2 - x - 12$$

$$0 = (x-4)(x+3)$$

~~$x = 4, -3$~~

$$c = -3$$

$$20.) f(x) = \begin{cases} 2x - 3ax^2, & x < 1 \\ bx - 4, & 1 \leq x \leq 2 \\ x^2 + a, & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1) \quad \left| \begin{array}{l} \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2) \\ \lim_{x \rightarrow 2^-} (bx - 4) = \lim_{x \rightarrow 2^+} (x^2 + a) \end{array} \right.$$

$$\lim_{x \rightarrow 1^-} (2x - 3ax^2) = \lim_{x \rightarrow 1^+} (bx - 4) \quad \left| \begin{array}{l} 2 - 3a = b - 4 \\ 2 - 3(2b - 8) = b - 4 \end{array} \right.$$

$$2 - 3a = b - 4 \quad -7b = -30 \quad \left| \begin{array}{l} -7b = -30 \\ b = \frac{30}{7} \\ 2 - 6b + 24 = b - 4 \end{array} \right.$$

$$2b - 4 = 4 + a$$

$$2b - 8 = a$$

$$\lim_{x \rightarrow \infty} \left(2 - \frac{5}{(x-1)^2} \right)$$

$$\lim_{x \rightarrow \infty} 2 - \lim_{x \rightarrow \infty} \frac{5}{(x-1)^2}$$

$$2 - 0$$

2

3.) $\lim_{x \rightarrow 0} \frac{\sin(x) + \cos(x)}{x^2}$

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1 + \cos x}{x} \right)$$

$1 \cdot D$
0

$$19.) f(x) = \begin{cases} 3x+a, & x \leq -3 \\ ax^2+4, & x > -3 \end{cases}$$

$$\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x) = f(-3)$$

$$\lim_{x \rightarrow -3^-} (3x+a) = \lim_{x \rightarrow -3^+} (ax^2+4)$$

$$-9+a = 9a+4$$

$$-13/8 = a$$

$$17) \lim_{x \rightarrow -6} \frac{|x+6|}{x+6} \text{ dne}$$

$$\lim_{x \rightarrow -6^-} \frac{|x+6|}{x+6} \neq \lim_{x \rightarrow -6^+} \frac{|x+6|}{x+6}$$

$$\frac{|-6.1+6|}{-6.1+6} \neq \frac{|-5.9+6|}{-5.9+6}$$
$$-1 \neq 1$$

$$y = \frac{x}{5x^2 - 10x}$$

$$y = \frac{x}{5x(x-2)}$$

Remov. $x = 0$

Nonremov. $x = 2$

$$\lim_{x \rightarrow -\infty} \frac{4x}{\sqrt{6x^2 + 1}}$$
$$= \frac{-4}{\sqrt{6}}$$