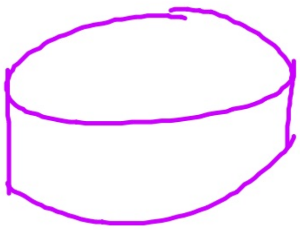
# 7.2: Volume by Revolution

two types

1) disk method

2) washer method

Disk Method (solid 2-D shape rotated around an axis)



$\pi \underline{r}^2 h$

$(3-7)^2 = (7-3)^2$

$$\pi \int_a^b r^2 \, dx$$

rotated about a horizontal axis

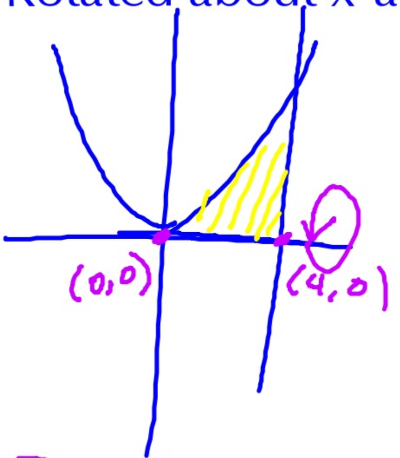
$$\pi \int_a^b r^2 \, dy$$

rotated about a vertical axis

r = function - axis

$r^2 = (\text{funct.} - \text{axis})^2$

1) Region bounded by: $y = x^2$, $x = 4$, $y = 0$
Rotated about x-axis




Disk

$r = x^2 - 0$

$r^2 = x^4$

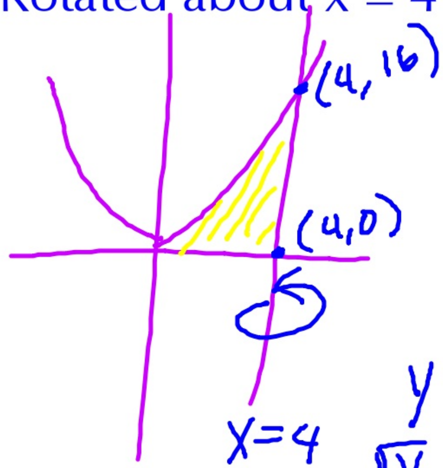
$$\pi \int_0^4 x^4 \, dx = \pi \frac{x^5}{5} \Bigg]_0^4$$

$$= \frac{4^5 \pi}{5}$$

$$\frac{1024\pi}{5}$$

$$\begin{array}{r} 256 \\ \times\ 4 \\ \hline 1024 \end{array}$$

2) Region bounded by: $y = x^2$, $x = 4$, $y = 0$
Rotated about $x = 4$

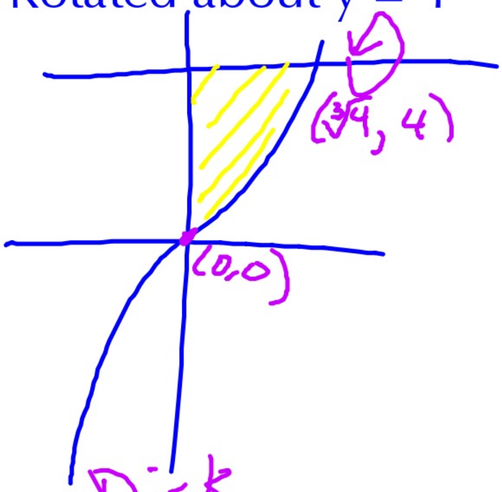



$$\pi \int_0^{16} \left(\sqrt{y} - 4\right)^2 dy$$

$y = x^2$
$\sqrt{y} = x$

Disk
$r = \left(\sqrt{y} - 4\right)$
$r^2 = \left(\sqrt{y} - 4\right)^2$

3) Region bounded by $y = x^3$, $y = 4$, $x = 0$.
Rotated about $y = 4$

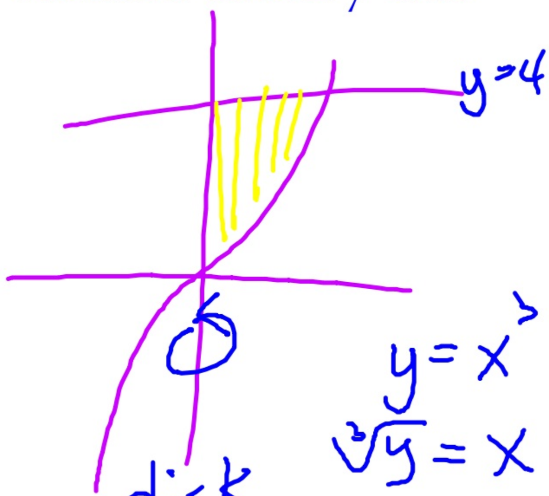



$$\pi \int_0^{\sqrt[3]{4}} (x^3 - 4)^2 \, dx$$

Disk

$r = (x^3 - 4)$

4) Region bounded by $y = x^3$, $y = 4$, $x = 0$.
Rotated about y-axis




$y = x^3$

$\sqrt[3]{y} = x$

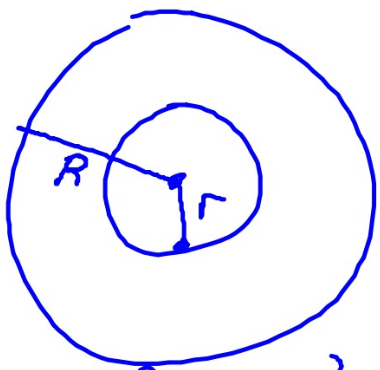
disk

$r = \sqrt[3]{y} - 0$

$$\pi \int_0^4 \left(\sqrt[3]{y}\right)^2 dy$$

$$\pi \int_0^4 y^{2/3} dy$$

Washers: When a 2-d shape is not completely connected to the axis of rotation



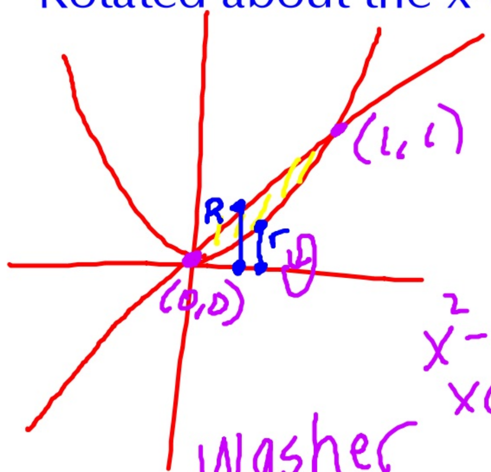

$$\pi R^2 - \pi r^2$$

$$\pi \int_a^b (\overset{R^2}{\text{outer} - \text{axis}})^2 - (\overset{r^2}{\text{inner} - \text{axis}})^2 \, d$$

or

$$\pi \int_a^b (\text{outer} - \text{axis})^2 - (\text{inner} - \text{axis})^2 \, d$$

5) Region bounded by $y = x^2$, $y = x$.
Rotated about the x-axis




$$\pi \int_0^1 R^2 - r^2 \, dx$$

$x^2 - x = 0$
$x(x-1) = 0$

Washer

$R = x - 0 \quad r = x^2 - 0$

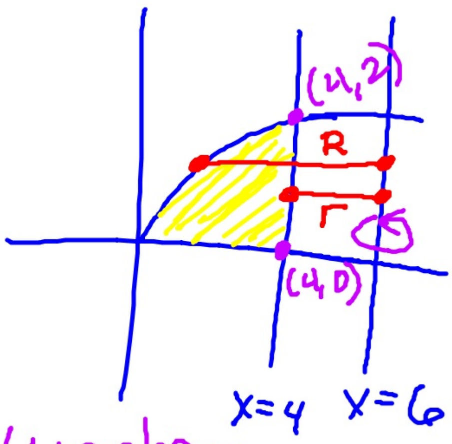
$R^2 = x^2 \quad r^2 = x^4$

$$\pi \int_0^1 x^2 - x^4 \, dx$$

$$\pi \left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1$$

$$\pi \left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$$

6) Region bounded by y = sqrt(x), y = 0, and x = 4.
Region rotated about x = 6




$$\pi \int_0^2 R^2 - r^2 \, dy$$

$y = \sqrt{x}$

$y^2 = x$

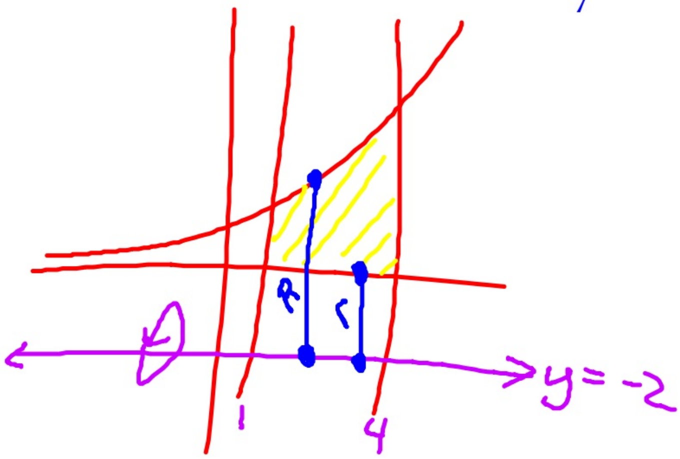
$$\pi \int_0^2 (y^2-6)^2 - (4) \, dy$$

washer

$R = y^2 - 6$

$r = 4 - 6 = -2$

7) Region bounded by $y = e^x$, $y = 0$, $x = 1$, $x = 4$.
Rotated about the line $y = -2$




$$\pi \int_1^4 \left( (e^x + 2)^2 - 4 \right) dx$$