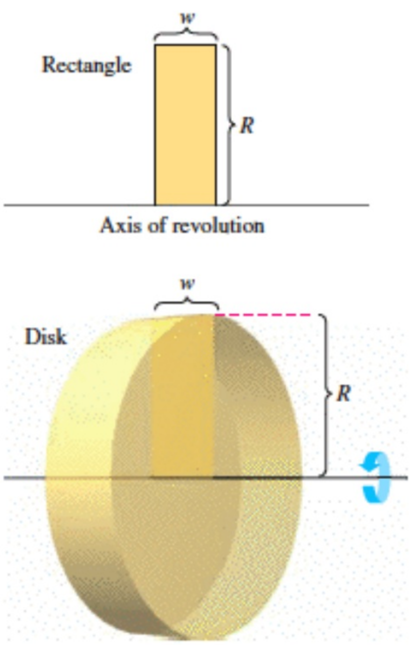
7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.



Volume of a disk: $\pi R^2 w$

Disk Method

$$V = \pi R^2 h$$

$$\pi \int_a^b R^2 \, dx \qquad R^2 = (\text{funct.} - \text{axis})^2$$

$$(3-7)^2 = (7-3)^2$$

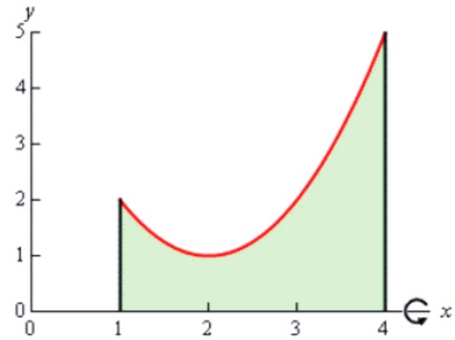
$$(\text{funct} - \text{axis})^2 = (\text{axis} - \text{funct.})^2$$

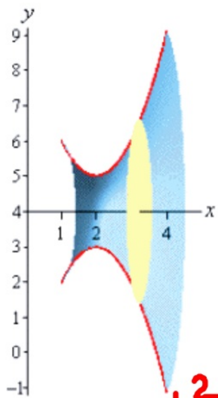
Volume by Revolution (Disk Method)

Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.

Set up only

$y = x^2 - 4x + 5$
$x = 1$, $x = 4$,
rotate about the x-axis



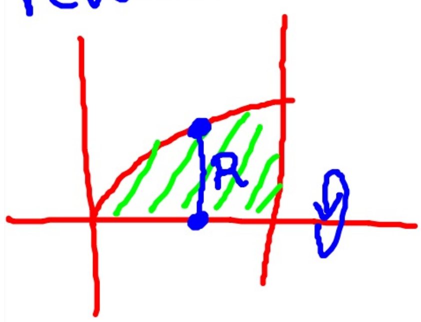


$$\pi\int_1^4 R^2\,dx = \pi\int_1^4 (x^2-4x+5-0)^2\,dx$$

① $y = \sqrt{x}$ $x = 4$

$y = 0$

revolve : x-axis



$$\pi \int_0^4 (\sqrt{x} - 0)^2 \, dx$$

$$\pi \int_0^4 x \, dx = \frac{\pi x^2}{2} \Bigg|_0^4$$

$$= 8\pi$$

When rotating around a horizontal axis, integrate with respect to x

② $y = \sqrt{x}$ $x = 4$ $\pi \int_0^2 (y^2 - 4)^2 \, dy$

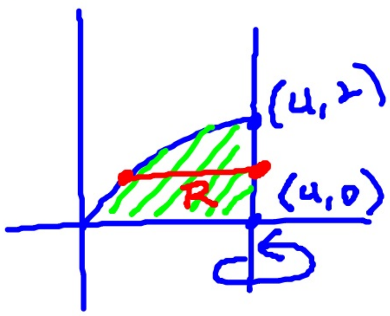
$y = 0$

revolve : $x = 4$

set up only

$y = \sqrt{x}$

$y^2 = x$



Rotating around a vertical axis: integrate with respect to y

(3) $y = x^2$, $x=2$

$y=0$

rotate : $x=2$

set up only

$$\pi\int_0^4 (\sqrt{y}-2)^2\,dy$$

Washer

$\pi R^2 - \pi r^2$

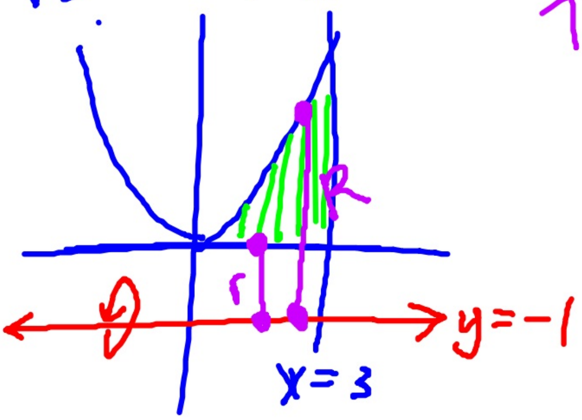
$R^2 - r^2$

$$\left(\begin{matrix}\text{outer}\\ \text{funct.}\end{matrix} - \text{axis}\right)^2 - \left(\begin{matrix}\text{inner}\\ \text{funct.}\end{matrix} - \text{axis}\right)^2$$

$$\pi \int_a^b \left(R^2 - r^2\right) dx$$

3 $y = x^2$, $y=0$, $x=3$ set up only

rotate: $y=-1$



washer

"x"

$$\pi\int_0^3 \left(R^2 - r^2\right)dx$$

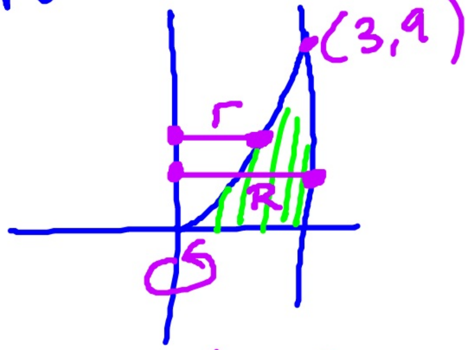
$$\pi\int_0^3 \left(\left(x^2-(-1)\right)^2 - \left(0--1\right)^2\right)dx$$

$$\pi\int_0^3 \left(\left(x^2+1\right)^2 - 1\right)dx$$

④ $y = x^2$, $x = 3$

$y = 0$

rotate: y-axis



washer

"y"

$$\pi\int_0^9 (R^2 - r^2)\,dy$$

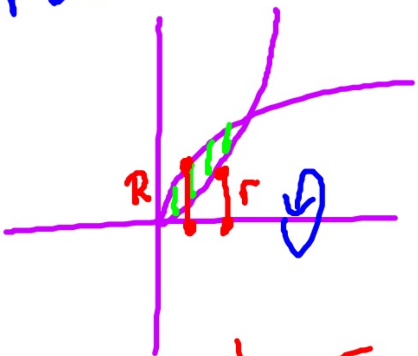
$$\pi\int_0^9 (3-0)^2 - (\sqrt{y}-0)^2\,dy$$

$$\pi\int_0^9 (9-y)\,dy$$

⑤ $y = x^2$

$y = \sqrt{x}$

rotate: x-axis



washer

"x"

$$\pi \int_0^1 (\sqrt{x} - 0)^2 - (x^2 - 0)^2 \, dx$$

$$\pi \int_0^1 (x - x^4) \, dx$$

Volume by Revolution (Washer Method)

$$V = \pi \int_a^b \left[R(x)\right]^2 - \left[r(x)\right]^2 dx$$

R^2 r^2

$$V = \pi \int_a^b \left[(outer - axis)^2 - (inner - axis)^2 \right] dx$$

Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.