7.2: Volume by cross section

You will be given:

- base (determined by region enclosed by functions)
- geometric shape
- perpendicular to x-axis or y-axis

$$V = \int_a^b \text{Area}$$

Geometric Shapes

Square S^2

Rectangle CS

C: constant

Equilateral triangle

$\frac{\sqrt{3}}{4} S^2$

Semi-circles

$\frac{\pi}{8} S^2$

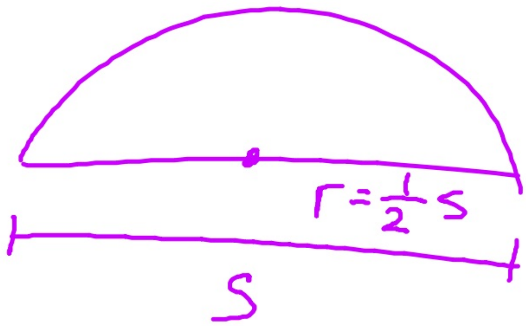
S = top - bottom

$\perp$ x-axis

S = right - left

$\perp$ y-axis

Semicircle

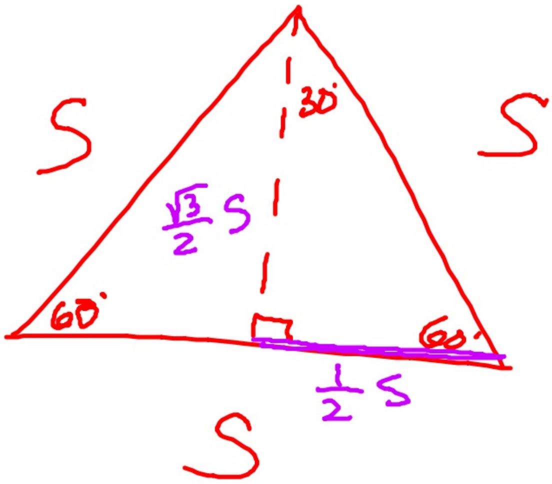


$$A = \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2}\pi\left(\frac{1}{2}s\right)^2$$

$$A = \frac{\pi}{8}s^2$$

Equilateral Δ



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right)$$

$$A = \frac{\sqrt{3}}{4}s^2$$

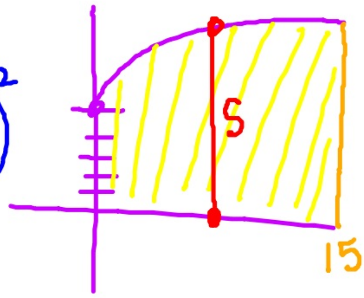
① $f(x) = 2\sqrt{x} + 5 \quad [0, 15] \quad 1^{ST}$ Quad.

Cross Sections perpendicular to the x-axis are semicircles

$V = \int \text{Area}$

$A = \frac{\pi}{8} s^2$

$\frac{\pi}{8}(2\sqrt{x} + 5 - 0)^2$



$$V = \int_0^{15} \frac{\pi}{8}(2\sqrt{x} + 5)^2 \, dx$$

② $f(x) = 2\sqrt{x} + 5 \quad [0, 15] \quad 1^{ST}$ Quad.

Cross sections perpendicular to the x-axis are squares

$$V = \int_0^{15} (2\sqrt{x}+5)^2 \, dx$$

$A = S^2$

$Area = (2\sqrt{x}+5)^2$

3) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the x-axis are squares.

$$\int_0^2 \text{Area}\, dx = \int_0^2 S^2\, dx$$

$$= \int_0^2 x^4\, dx = \frac{1}{5}x^5 \Big]_0^2$$

$$= \frac{32}{5}$$

$$S = x^2 - 0$$

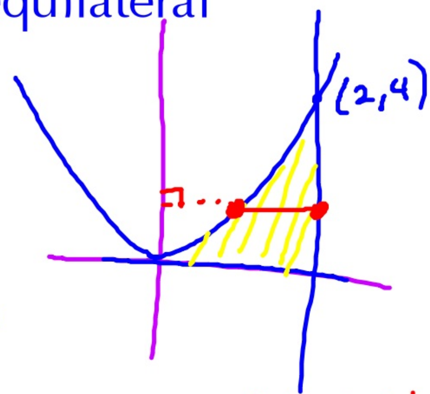
$$S^2 = x^4$$

4) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the y-axis are equilateral triangles.

$$\int_0^4 \frac{\sqrt{3}}{4} S^2 \, dy$$

$$V = \frac{\sqrt{3}}{4}\int_0^4 \left(2-\sqrt{y}\right)^2 dy$$

$$y = x^2$$
$$\pm\sqrt{y} = x$$
$$\sqrt{y} = x$$

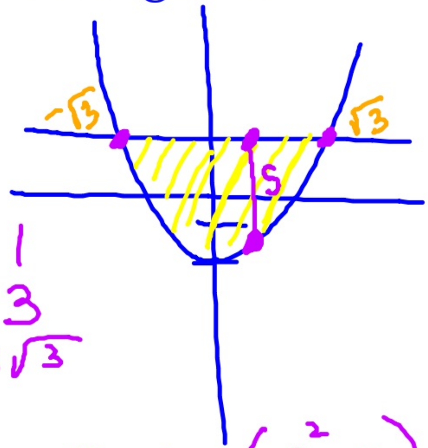


S = right - left

$S = 2 - \sqrt{y}$

5) base enclosed by $y = x^2 - 2$ and $y = 1$. Cross sections perpendicular to the x-axis are rectangles of height 5.

$$= \int_{-\sqrt{3}}^{\sqrt{3}} 5s\,dx$$



$$\frac{\sqrt{27}}{3} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$$x^2 - 2 = 1$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

$$= 5\int_{-\sqrt{3}}^{\sqrt{3}} (3 - x^2)\,dx = 5\left(3x - \frac{1}{3}x^3\right)\Big]_{-\sqrt{3}}^{\sqrt{3}}$$

$$5\left[(3\sqrt{3} - \sqrt{3}) - (-3\sqrt{3} + \sqrt{3})\right]$$

$$5(6\sqrt{3} - 2\sqrt{3}) = 20\sqrt{3}$$

$$S = 1 - (x^2 - 2)$$
$$S = 3 - x^2$$