# 7.2: Volume by cross section

You will be given:

- base (determined by region enclosed by functions)
- geometric shape
- perpendicular to x-axis or y-axis

# Geometric Shapes

Square $A = S^2$

Rectangle $A = C \cdot S$

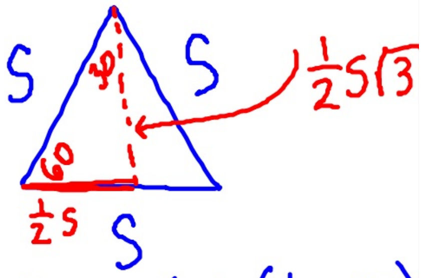
$C$ : constant

Equilateral triangle

$A = \frac{\sqrt{3}}{4} S^2$

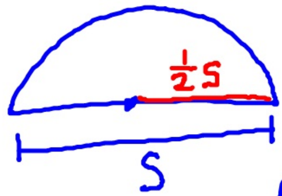
Semi-circles

$A = \frac{\pi}{8} S^2$




$$A = \frac{1}{2}(S)\left(\frac{1}{2} S\sqrt{3}\right)$$

$$A = \frac{\sqrt{3}}{4} S^2$$




$$A = \frac{1}{2}\pi\left(\frac{S}{2}\right)^2$$

$$A = \frac{\pi}{8} S^2$$

**Perpendicular to the x-axis**

$$V = \int_a^b (\text{Area})\,dx$$

in terms of x

**Perpedicular to the y-axis**

$$V = \int_a^b (\text{Area})\,dy$$

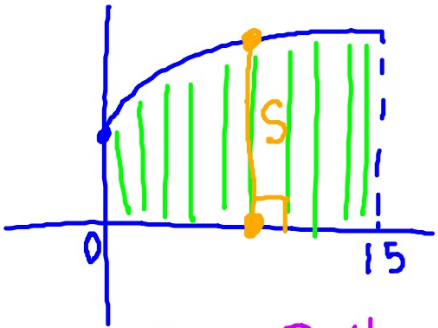
in terms of y

1 $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

***Cross section perpendicular to the x-axis are squares. (set up only)***

$$V = \int_0^{15} S^2 \, dx$$

$$V = \int_0^{15} (2\sqrt{x} + 5)^2 \, dx$$




$S = \text{Top} - \text{Bottom}$

$S = 2\sqrt{x} + 5 - 0$

② $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

***Cross section perpendicular to the x-axis aresemi-circles. (set up only)***

$$V = \int_0^{15} \frac{\pi}{8} s^2 \, dx = \frac{\pi}{8} \int_0^{15} (2\sqrt{x} + 5)^2 \, dx$$

③ $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

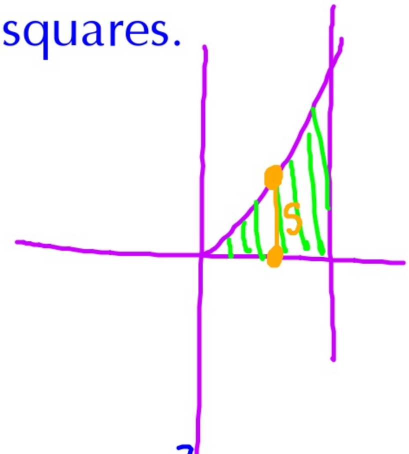
***Cross section perpendicular to the x-axis are rectangles of height 4. (set up only)***

$$V = \int_0^{15} 4s\,dx = \int_0^{15} 4(2\sqrt{x} + 5)\,dx$$

4) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the x-axis are squares.

$$V = \int_0^2 s^2\,dx = \int_0^2 x^4\,dx$$

$$= \left.\frac{x^5}{5}\right|_0^2 = \frac{32}{5}$$



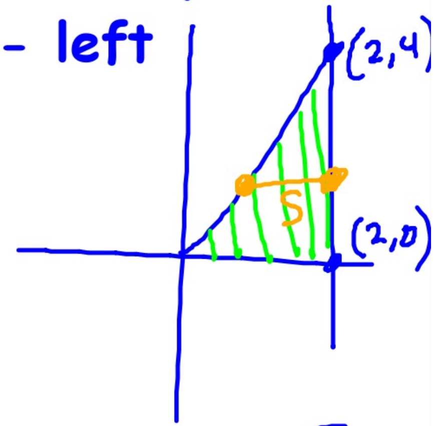

$$s = x^2 - 0$$

**perpendicular to x-axis**
**S = top - bottom**

5) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the y-axis are equilateral triangles. (set up only)

**perpendicular to y-axis:**

**S = right - left**




$$y = x^2$$

$$\pm\sqrt{y} = x$$

$$\sqrt{y} = x$$

$$\int_0^4 \frac{\sqrt{3}}{4} s^2 \, dy$$

$$= \frac{\sqrt{3}}{4}\int_0^4 (2-\sqrt{y})^2 \, dy$$

$$S = 2 - \sqrt{y}$$