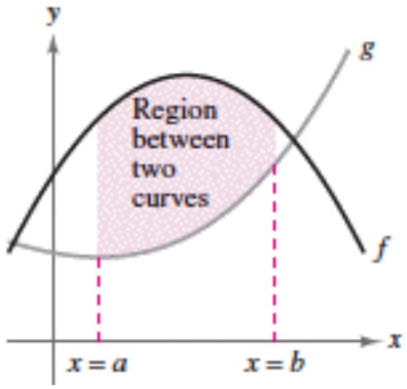
Quiz: Monday

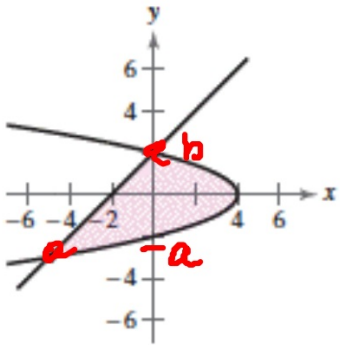
Pg. 454: 19 - 25 odd, 49

set up only: 1, 3, 5, 17b, 29, 31, 52

7.1 Area of a Region Between Two Curves

- Find the area of a region between two curves using integration.
- Find the area of a region between intersecting curves using integration.





Top - Bottom

$$\int_a^b (f(x) - g(x))\,dx$$

Right - Left

$$\int_a^b (\text{parabola} - \text{line})\,dy$$

#1

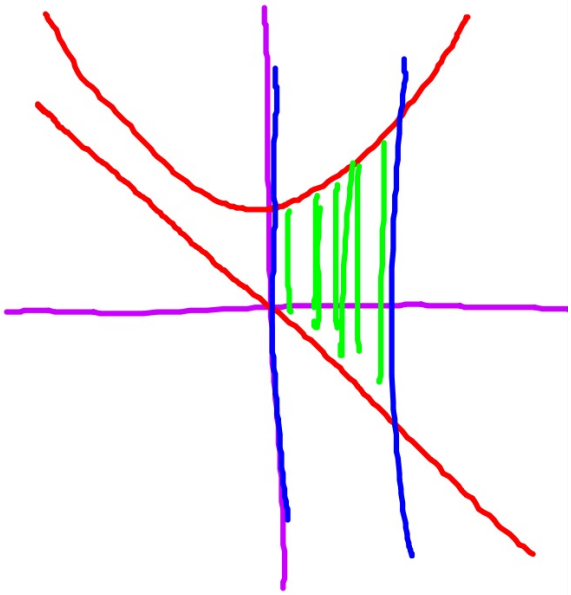
Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

$y = x^2 + 2,\ y = -x,\ x = 0,\ x = 1$

$$\int_0^1 (x^2 + 2 - (-x))\,dx$$

$$\frac{x^3}{3} + 2x + \frac{x^2}{2}\Big|_0^1$$

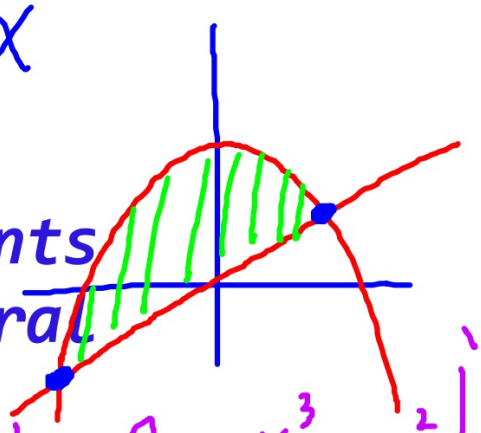
$$\frac{1}{3} + 2 + \frac{1}{2} = 2\frac{5}{6}$$



#2 Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.

$f(x) = 2 - x^2 \quad g(x) = x$

1) sketch and shade
2) find intersection points
3) set up definite integral and evaluate



$2 - x^2 = x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2, 1$

$$\int_{-2}^{1} (2 - x^2 - x)\,dx = 2x - \frac{x^3}{3} - \frac{x^2}{2}\Big|_{-2}^{1}$$

$$\left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right)$$

$2 + 4 + 2 \qquad -\frac{1}{3} - \frac{8}{3}$

$8 \qquad -3 \qquad -\frac{1}{2} = 8 - 3 - \frac{1}{2}$

$4\frac{1}{2}$

#4 $f(y) = y(2 - y), \quad g(y) = -y$

$x = y(2-y)$

x	y
0	2
0	0
1	1

$x = -y$

$-x = y$

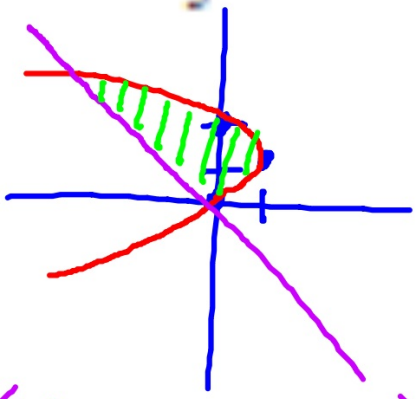
$y(2-y) = -y$

$2y - y^2 = -y$

$3y - y^2 = 0$

$y(3-y) = 0$

$y = 0, 3$



$\int_0^3 (y(2-y) - (-y))\,dy$

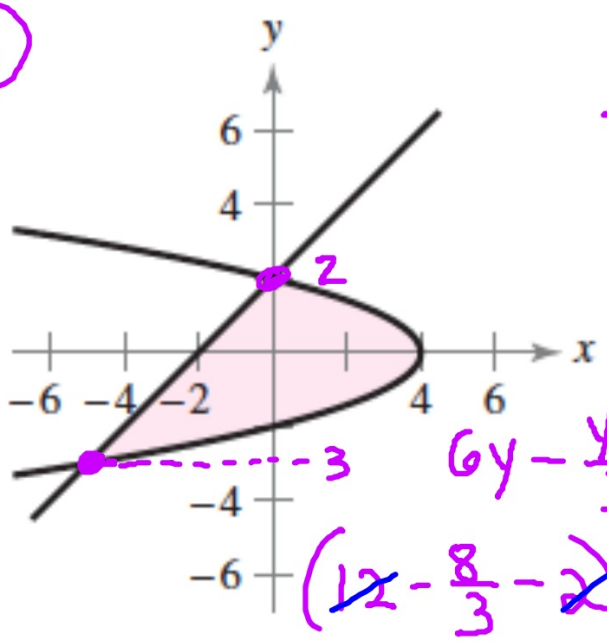
$\int_0^3 (2y - y^2 + y)\,dy$

$\int_0^3 (3y - y^2)\,dy$

$x = 4 - y^2$

$x = y - 2$

(3)



Right - left

$$\int_{-3}^{2} \left(4 - y^2 - (y-2)\right) dy$$

$$\int_{-3}^{2} (6 - y^2 - y)\, dy$$

$$6y - \frac{y^3}{3} - \frac{y^2}{2} \Big|_{-3}^{2}$$

$$\left(12 - \frac{8}{3} - 2\right) - \left(-18 + 9 - \frac{9}{2}\right)$$

$$19 - \frac{8}{3} + \frac{9}{2} = 19 + \frac{-16+27}{6} = \frac{125}{6} \text{ or } 20\frac{5}{6}$$