6.3: Separation of Variables and differential equations

Solve all equations for "y"

For example: $\ln y = x + C$ is not the final answer.

The final answer is: $y = Ce^x$

$$e^{\ln y} = e^{x + C}$$

$$y = e^x \cdot e^C \rightarrow C$$

$$y = Ce^x$$

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① $\frac{dy}{dx} = \frac{3y}{x^2}$

Find a general solution for the differential equation.

$$\int \frac{dy}{y} = \int \frac{3}{x^2}\,dx$$

$$\ln|y| = 3\int x^{-2}\,dx$$

$$\ln|y| = \frac{3 \cdot x^{-1}}{-1} + C$$

$$e^{\ln|y|} = e^{-\frac{3}{x} + C}$$

$$|y| = Ce^{-3/x}$$

or

$$y = Ce^{-3/x}$$

(2) $\sqrt{1-x^2}\,y' - x = 0$

$$\sqrt{1-x^2}\,\frac{dy}{dx} - x = 0$$

$$\frac{dy}{dx}\sqrt{1-x^2} = x$$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = dx$

$$\int dy = \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$y = -\frac{1}{2}\int u^{-1/2}\,du$$

$$y = -\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C$$

$$y = -u^{1/2} + C$$

$$y = -\sqrt{1-x^2} + C$$

Find the particular solution to the differential equation.

3 $\quad y' - (x+3)y^2 = 0 \qquad (0,1)$

$$\frac{dy}{dx} = (x+3)y^2$$

$$\int \frac{dy}{y^2} = \int (x+3)\,dx$$

$$\int y^{-2}\,dy = \frac{x^2}{2} + 3x + C$$

$$-\frac{1}{y} = \frac{x^2}{2} + 3x + C$$

find C

$-1 = 0 + 0 + C$

$-1 = C$

$$-\frac{1}{y} = \frac{x^2}{2} + 3x - 1$$

$$\frac{1}{y} = -\frac{x^2}{2} - 3x + 1$$

$$\frac{1}{y} = \frac{-x^2 - 6x + 2}{2}$$

$$y = \frac{2}{-x^2 - 6x + 2}$$

Find the general solution to the differential equation. $\ln e^x = x$

4 $y' - e^y \cos x = 0$

$$\frac{dy}{dx} = e^y \cos x$$

$$\int \frac{dy}{e^y} = \int \cos x \, dx$$

$$\int e^{-y} dy = \sin x + C$$

$u = -y$
$du = -1 dy$

$$-1\int e^u du = \sin x + C$$

$$-e^u = \sin x + c$$

$$-e^{-y} = \sin x + C$$

$$\ln(e^{-y}) = \ln(-\sin x + C)$$

$$-y = \ln|-\sin x + c|$$

$$y = -\ln|-\sin x + c|$$

Find the particular solution to the differential equation given the point (1, 2)

5. $y^2 y' - x^2 = 0$

$\int y^2 \frac{dy}{\cancel{dx}} = \int x^2 dx$

$\frac{1}{3} y^3 = \frac{1}{3} x^3 + C$

(1, 2) $\frac{8}{3} = \frac{1}{3} + C$

$\frac{7}{3} = C$

$\underline{C = 7/3}$

$3\left(\frac{1}{3} y^3 = \frac{1}{3} x^3 + \frac{7}{3}\right)$

$\sqrt[3]{y^3} = \sqrt[3]{x^3 + 7}$

$y = \underline{\sqrt[3]{x^3 + 7}}$

Find the particular solution to the differential equation given (1, 2)

6 $\quad yy' + x = 0$

$$y\frac{dy}{dx} = -x$$

$$\int y\,dy = \int -x\,dx$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$$

find C

$$2 = -\frac{1}{2} + C$$

$$\frac{5}{2} = C$$

$$2\left(\frac{1}{2}y^2 = -\frac{1}{2}x^2 + \frac{5}{2}\right)$$

$$\sqrt{y^2} = \sqrt{-x^2 + 5}$$

$$y = \pm\sqrt{-x^2 + 5}$$

$$y = \sqrt{-x^2 + 5}$$