6.3: Separation of Variables/Differential Equations

(a) $\frac{dP}{dt} = \frac{K}{t^2}$

$\int dP = \int Kt^{-2}\,dt$

$P = \frac{Kt^{-1}}{-1} + C$

$P = -\frac{K}{t} + C$; $P = \frac{K}{t} + C$

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19.) $\frac{dy}{dt} = -\frac{1}{2}y$ $\quad (0, 10)$

$\int \frac{dy}{y} = \int -\frac{1}{2}\,dt$

$\ln|y| = -\frac{1}{2}t + C$

$\ln 10 = C$

$\ln|y| = -\frac{1}{2}t + \ln 10$

$e^{\ln|y|} = e^{-\frac{1}{2}t + \ln 10}$

$y = e^{-\frac{1}{2}t} \cdot e^{\ln 10}$

$\boxed{y = 10e^{-\frac{1}{2}t}}$

$x^{2+3} = x^2 \cdot x^3$

Solve the diff. eq.

① $\frac{dy}{dx} = \frac{3y}{x^2}$

$\int \frac{dy}{y} = \int 3x^{-2}\,dx$

$\ln|y| = \frac{3x^{-1}}{-1} + C$

$e^{\ln|y|} = e^{-\frac{3}{x} + C}$

$y = e^{-3/x} \cdot e^{C}$

$y = Ce^{-3/x}$

(2) $yy' - x = 0$

$y\frac{dy}{dx} = x$

$\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$2 = C$

$(0,2)$

$2\left(\frac{1}{2}y^2 = \frac{1}{2}x^2 + 2\right)$

$\sqrt{y^2} = \sqrt{x^2+4}$

$y = \pm\sqrt{x^2+4}$

↑ pick one!

$y = \sqrt{x^2+4}$ ← because $(0,2)$ works!

③ $y' - e^y \cos x = 0$

$\frac{dy}{dx} = e^y \cos x$

$\frac{dy}{e^y} = \cos x \, dx$

$\int e^{-y} dy = \int \cos x \, dx$

$u = -y$

$du = -1 dy$

$-du = dy$

$-\int e^u du$

$-e^{-y} = \sin x + C$

$\ln|e^{-y}| = \ln|-\sin x + C|$

$-y = \ln|-\sin x + C|$

$y = -\ln|-\sin x + C|$

$$\frac{1}{y} = \frac{x}{2} + 5$$

$$\frac{1}{y} = \frac{x+10}{2}$$

$$y = \frac{2}{x+10}$$

$$y = \frac{1}{\frac{x}{2} + 5}$$

(4) $\frac{dy}{y^2} = (x+3)dx$ $(0,1)$

$\int y^{-2}dy = \int (x+3)dx$

$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 3x + C$

$-1 = 0 + 0 + C$

$-1 = C$

$-y^{-1} = \frac{x^2}{2} + 3x - 1$

$\frac{1}{y} = -\frac{x^2}{2} - 3x + 1$

$\frac{1}{y} = \frac{-x^2 - 6x + 2}{2}$

$y = \frac{2}{-x^2 - 6x + 2}$