

6.2: Direct and Inverse Variation

$$1.) g = kh$$

k : constant of proportionality

$$2.) r = \frac{k}{t}$$

$$5.) y = kxz$$

$$3.) x = ky^3$$

$$4.) a = \frac{k}{\sqrt{b}}$$

$$6.) C = KM$$

$$400 = K(3000)$$

$$\frac{2}{15} = K$$

$$C = \frac{2}{15}M$$

$$M = 3000$$

$$C = 400$$

7) The rate of change of y is (directly)proportional to y.

$$\frac{dy}{dx} = ky$$

$$\int \frac{dy}{y} = \int k dx$$

$$e^{\ln|y|} = e^{kx + C}$$

$$y =$$

$$|y| = e^{kx + C}$$

$$|y| = Ce^{kx}$$

$$y = Ce^{kx}$$

$$e^{kx + C} = e^{kx} \cdot e^C$$

The rate of change of P with respect to t is proportional to (10 - t)

$$\frac{dP}{dt} = k(10-t)dt$$

$$\int dP = \int k(10-t)dt$$

$$P = k\left(10t - \frac{1}{2}t^2\right) + C$$

The rate of change of y is inversely proportional to x

$$9.) \quad \frac{dy}{dx} = \frac{k}{x}$$

$$\int dy = \int \frac{k}{x} dx$$

$$y = k \ln|x| + C$$

Find the function $y = f(t)$ passing through the point $(0, 10)$ with the given derivative.

$$e^{\ln x} = x$$

$$12.) \quad \frac{dy}{dt} = \frac{3}{4}y$$

$$\int \frac{dy}{y} = \int \frac{3}{4} dt$$

$$\ln|y| = \frac{3}{4}t + C$$

$$\ln 10 = 0 + C$$

$$\ln 10 = C$$

$$e^{\ln y} = e^{\frac{3}{4}t + \ln 10}$$

$$y = e^{\frac{3}{4}t} (e^{\ln 10})$$

$$y = 10e^{\frac{3}{4}t}$$

$$10.) N = Ce^{kt}$$

$$250 = Ce^0$$

$$250 = C$$

$$N = 250e^{kt}$$

$$400 = 250e^k$$

$$\ln \frac{8}{5} = \ln e^k$$

$$(0, 250) \quad (1, 400)$$

$$\ln \frac{8}{5} = k$$

$$\ln e^k$$

$$k \cdot \ln e$$

$$k \cdot 1$$

$$k$$

$$N = 250e^{\ln \frac{8}{5} t}$$

or

$$N = 250 \left(e^{\ln \frac{8}{5}} \right)^t = 250 \left(\frac{8}{5} \right)^t$$

Find the exponential function $y = Ce^{kt}$ given $(3, 1/2)$ and $(4, 5)$.

$$e^{\ln x} = x$$

$$\frac{1}{2} = Ce^{3k}$$

$$\frac{\frac{1}{2}}{e^{3k}} = C$$

$$\frac{\frac{1}{2}}{e^{3 \ln 10}} = C$$

$$\frac{\frac{1}{2}}{e^{\ln 1000}} = C = \frac{1}{1000}$$

$$5 = Ce^{4k}$$

$$5 = \frac{1}{2} e^{3k} \cdot e^{4k}$$

$$5 = \frac{1}{2} e^k$$

$$10 = e^k$$

$$\ln 10 = k$$

$$y = \frac{1}{2000} e^{(\ln 10)t}$$

$$y = \frac{1}{2000} (10)^t$$