18.) $\frac{dy}{dt} = -\frac{3}{4}\sqrt{t}$ $(0, 10)$

$\int dy = \int -\frac{3}{4} \cdot t^{1/2} dt$

$y = -\frac{3}{4} \cdot \frac{t^{3/2}}{3/2} + C$

$y = -\frac{1}{2} t^{3/2} + C$

$10 = C$

$\boxed{y = -\frac{1}{2} t^{3/2} + 10}$

4.) $\frac{dy}{dx} = (6-y)$

$$\int \frac{dy}{6-y} = \int dx$$

$u = 6-y$

$du = -1dy$

$$-\int \frac{1}{u}du$$

$$-\ln|6-y| = x + C$$

$$e^{\ln|6-y|} = e^{-x+C}$$

$$6-y = e^{-x+c}$$

$$6-y = e^{-x} \cdot e^{c}$$

$$6-y = Ce^{-x}$$

$$6-Ce^{-x} = y$$

47.) $\frac{dy}{dx} = \sin 2x$

$\int dy = \int \sin 2x \, dx$

$u = 2x$
$du = 2\,dx$
$\frac{du}{2} = dx$

$y = \int \sin u \cdot \frac{du}{2}$

$y = \frac{1}{2}\int \sin u \, du$

$y = -\frac{1}{2}\cos 2x + C$

6.2: Differential Equations/Proportionality

The rate of change of y is directly proportional to t.

multiply

k: constant of proportionality

$$\frac{dy}{dt} = kt$$

$$\int dy = \int kt\,dt \;;\; y = \frac{kt^2}{2} + C = kt^2 + C$$

The rate of change of y is inversely proportional to the cube root of t

divide

$$\frac{dy}{dt} = \frac{k}{\sqrt[3]{t}}$$

$$\int dy = \int kt^{-1/3}\,dt$$

$$y = \frac{k \cdot t^{2/3}}{2/3} + C$$

$$y = kt^{2/3} + C \quad \text{or} \quad y = \frac{3}{2}kt^{2/3} + C$$

The rate of change of y is proportional to y. implied directly

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$e \qquad e$$

$$y = e^{kt} \cdot e^{C} = Ce^{kt}$$

$$y = Ce^{kt}$$

The rate of change of y is proportional to y. $y = Ce^{kx}$

If y = 3 when x = 0 and y = 24 when x = 3. Find y when x = 5.

$(0,3)$ $(3,24)$

$3 = Ce^{0}$ $24 = 3e^{3k}$

$3 = C$ $\ln 8 = \ln e^{3k}$

$\ln 8 = 3k$

$\frac{1}{3}\ln 8 = k$

$\ln 2 = k$

$y = 3e^{\ln 2 \cdot x}$

$y = 3(e^{\ln 2})^{x}$

$y = 3(2^{x})$

$y = 3(2^{5}) = 96$ $e^{\ln x} = x$

Write an exponential function $y = a(b)^x$ given (0, 10) and (4, 5/8)

$(0, 10)$ $\quad (4, \frac{5}{8})$

$10 = a \cdot b^0$

$10 = a$

$\frac{5}{8} = 10 \cdot b^4$

$\frac{5}{80} = b^4$

$\frac{1}{16} = b^4$

$\frac{1}{2} = b$

$y = 10\left(\frac{1}{2}\right)^x$

$y = Ce^{kx}$

$(0, 10)$ $\quad (4, \frac{5}{8})$

$10 = Ce^0$

$10 = C$

$\frac{5}{8} = 10e^{k \cdot 4}$

$\ln \frac{1}{16} = \cancel{\ln} e^{4k}$

$\ln \frac{1}{16} = 4k$

$\frac{1}{4} \ln \frac{1}{16} = k$

$\ln \frac{1}{2} = k$

$y = 10e^{\ln \frac{1}{2} x}$

$y = 10\left(\frac{1}{2}\right)^x$

Write an exponential function $y = a(b)^x$ given (1, 12) and (3, 108)

$(1, 12)$ $(3, 108)$

$12 = a \cdot b^1$

$\frac{12}{b} = a$

$4 = a$

$108 = a \cdot b^3$

$108 = \frac{12}{b} \cdot b^3$

$108 = 12b^2$

$9 = b^2$

$3 = b$

$y = 4(3)^x$

Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$