Find the derivative.

① $y = \ln \dfrac{x^2}{\sqrt{x+1}}$

$y = 2\ln x - \frac{1}{2}\ln(x+1)$

$y' = \dfrac{2}{x} - \dfrac{1}{2(x+1)}$

② $y = xe^{x^2}$

$y' = x \cdot e^{x^2} \cdot 2x + e^{x^2} \cdot 1$

$y' = e^{x^2}(2x^2+1)$

③ $y = \log_4 (x^2+1)^3$

$y = 3\log_4 (x^2+1)$

$y' = 3 \cdot \dfrac{1}{\ln 4} \cdot \dfrac{2x}{x^2+1}$

④ $y = x5^{2x}$

$y' = x \cdot \ln 5 \cdot 5^{2x} \cdot 2 + 5^{2x} \cdot 1$

$y' = 5^{2x}(2x\ln 5 + 1)$

Find the derivative.

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$= e^{x^2}(2x^2+1)$

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$= 5^{2x}(2x\ln 5 + 1)$

(5) $y = (1+\ln x)^4$

$y' = 4(1+\ln x)^3 \cdot \frac{1}{x}$

or

$y' = \frac{4(1+\ln x)^3}{x}$

(6) $y = \frac{e^x}{e^x+1}$

$y' = \frac{(e^x+1)e^x - e^x(e^x)}{(e^x+1)^2}$

$y' = \frac{e^x}{(e^x+1)^2}$

Find the derivative.

① $y = e^{2x}$

② $y = \log_2\left(\frac{x}{x^4+1}\right)$

③ $y = 5^{2x}$

④ $y = \ln\sqrt{x-2}$

Evaluate.

⑤ $\int \frac{1}{x-2}\,dx$

⑥ $\int e^{4x}\,dx$

⑦ $\int 2^x\,dx$

⑧ $\int \cot x\,dx$

Find the derivative.

1. $y = e^{2x}$ $y' = 2e^{2x}$

2. $y = \log_2\left(\frac{x}{x^4+1}\right)$ $y' = \frac{1}{\ln 2}\left(\frac{1}{x} - \frac{4x^3}{x^4+1}\right)$

3. $y = 5^{2x}$ $y' = \ln 5 \cdot 5^{2x} \cdot 2$

4. $y = \ln\sqrt{x-2}$

   $y' = \frac{1}{2(x-2)}$

Evaluate.

5. $\int \frac{1}{x-2}\,dx = \ln|x-2| + C$

6. $\int e^{4x}\,dx = \frac{1}{4}e^{4x} + C$

7. $\int 2^x\,dx = \frac{1}{\ln 2}\cdot 2^x + C$

8. $\int \cot x\,dx = \ln|\sin x| + C$

$$\frac{x}{x-1} - \frac{1}{x}$$

$$\frac{x^2 - (x-1)}{x(x-1)}$$

$$\frac{x^2 - x + 1}{x(x-1)}$$

$$\frac{x}{x+1} + \frac{-1}{x-1}$$

$$2\ln 5 = \ln 25$$

$$\frac{x(x-1) + -1(x+1)}{(x+1)(x-1)}$$

$$\frac{x^2 - x - x - 1}{(x+1)(x-1)} = \frac{x^2 - 2x - 1}{(x+1)(x-1)}$$

Applications for derivatives

Equation of tangent line
Relative extrema, POI
Implicit differentiation

Applications for anti-deriv.

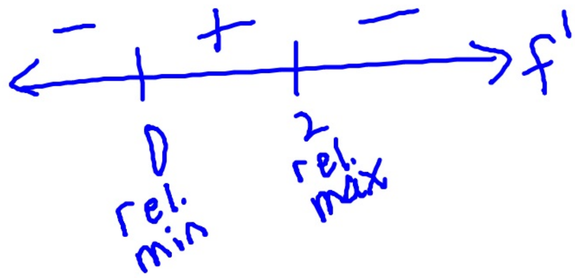
Average value
finding area
differential equations

rel. extrema

$$f(x) = x^2 e^{-x}$$

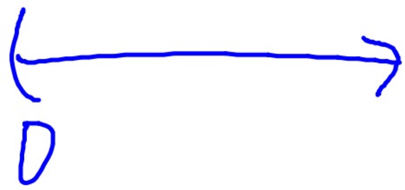
$$f'(x) = x^2 e^{-x}(-1) + e^{-x} \cdot 2x$$

$$= -xe^{-x}(x-2)$$



POI : $y = x - \ln x$

Domain: $(0,\infty)$



$y' = 1 - \frac{1}{x}$

$y'' = 0 + x^{-2}$

$y'' = \frac{1}{x^2}$

no POI ☺

15.) $y = \ln(2+x) + 2(2+x)^{-1}$

$$y' = \frac{1}{2+x} - 2(2+x)^{-2} \cdot 1$$

$$y' = \frac{1}{2+x} - \frac{2}{(2+x)^2}$$

$$y'(-1) = 1 - \frac{2}{1}$$

$$= -1$$

$(-1, 2)$ $m = -1$

$y - 2 = -1(x+1)$

$y = 2^{2x}$

$y' = \ln 2 \cdot 2^{2x} \cdot 2$

$= 2\ln 2 \cdot 2^{2x}$

$= \ln 4 \cdot 2^{2x}$

$\int 2^{2x}\,dx$

$u = 2x$
$du = 2dx$

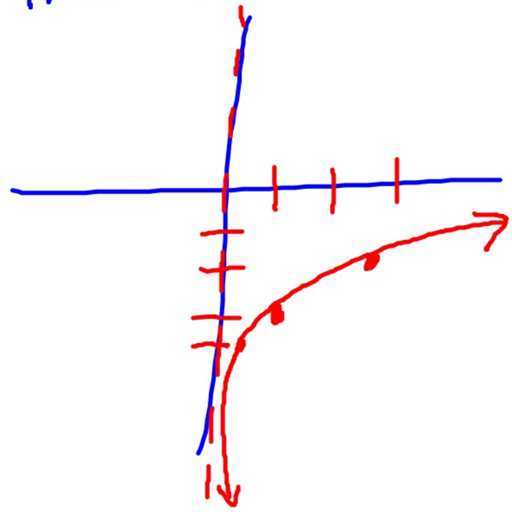
$\frac{1}{2}\int 2^u\,du$

$\frac{1}{2} \cdot \frac{1}{\ln 2} \cdot 2^u + C$

$\frac{1}{2\ln 2} \cdot 2^{2x} + C$

① $f(x) = \ln x - 3$

x	y
1	-3
e	-2
$1/e$	-4



2^x

e^x

2^{-x}

e^{-x}

$\frac{1}{x}$

~~ln x~~

11.) $f(x) = x\sqrt{\ln x}$

$$f(x) = x(\ln x)^{1/2}$$

$$f'(x) = x \cdot \frac{1}{2}(\ln x)^{-1/2} \cdot \frac{1}{x} + (\ln x)^{1/2} \cdot 1$$

$$= \frac{1}{2\sqrt{\ln x}} + \frac{\sqrt{\ln x}}{1}$$

$$= \frac{1 + 2\ln x}{2\sqrt{\ln x}}$$

$$f'(e) = \frac{3}{2}$$

21.) $\int_1^4 \frac{2x+1}{2x}dx = \int_1^4 \frac{2x}{2x}dx + \int_1^4 \frac{1}{2x}dx$

$= \int_1^4 1\,dx + \frac{1}{2}\int_1^4 \frac{1}{x}dx = x + \frac{1}{2}\ln|x| \Big|_1^4$

$\left(4+\frac{1}{2}\ln 4\right) - \left(1 + \cancel{\frac{1}{2}\ln 1}\right)$

$3 + \frac{1}{2}\ln 4 = 3 + \ln 2$

3.) $y = \ln \sqrt[5]{\dfrac{4x^2-1}{4x^2+1}}$

$$y = \frac{1}{5}\ln(4x^2-1) - \frac{1}{5}\ln(4x^2+1)$$

$$y = \frac{1}{5}\ln(2x+1) + \frac{1}{5}\ln(2x-1) - \frac{1}{5}(4x^2+1)$$

$$y' = \frac{1}{5}\cdot\frac{2}{2x+1} + \frac{1}{5}\cdot\frac{2}{2x-1} - \frac{1}{5}\cdot\frac{8x}{4x^2+1}$$

7

$$e^{\ln\sqrt{x+1}} = e^{2}$$

$$\sqrt{x+1} = e^{2}$$

$$x+1 = e^{4}$$

$$x = e^{4} - 1$$

69.) $g(x) = \log_3 \sqrt{1-x}$

$$g(x) = \frac{1}{2}\log_3(1-x)$$

$$g'(x) = \frac{1}{2} \cdot \frac{1}{\ln 3} \cdot \frac{-1}{1-x}$$

$$\frac{1}{2\ln 3} \cdot \frac{1}{x-1}$$

$$\frac{1}{\ln 9(x-1)}$$

71.) $\int (x+1)5^{(x+1)^2}dx$

$u=(x+1)^2$

$du=2(x+1)dx$

$\frac{du}{2(x+1)}=dx$

$\frac{1}{2}\int 5^u du$

$\frac{1}{2}\cdot\frac{1}{\ln 5}\cdot 5^u + C$

$\frac{1}{2\ln 5}5^{(x+1)^2}+C$

49.) $\int_0^1 x e^{-3x^2} dx$

$u = -3x^2$

$du = -6x\,dx$

$-\frac{1}{6}\int e^u du = -\frac{1}{6} e^{-3x^2} \Big|_0^1$

$-\frac{1}{6}\left(e^{-3} - e^0\right) = -\frac{1}{6}\left(e^{-3} - 1\right)$

$\frac{1}{6} - \frac{1}{6}e^{-3}$

113.) p. 360

$$\int \frac{5-e^x}{e^{2x}}dx = \int 5e^{-2x}dx - \int e^{-x}dx$$

$u=-2x$
$du=-2dx$

$u=-x$
$du=-1dx$

$$-\frac{5}{2}e^{-2x} + e^{-x} + C$$

119.)

$$\int_0^1 xe^{-x^2}dx = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^{-x^2}\Big|_0^1$$

$u=-x^2$
$du=-2xdx$

$$-\frac{1}{2}(e^{-1}-1) = -\frac{1}{2}\left(\frac{1}{e}-1\right)$$

51.) $\int \frac{e^{4x} - e^{2x} + 1}{e^{x}} dx$

$\int \left(e^{3x} - e^{1x} + e^{-x} \right) dx$

$\frac{1}{3} e^{3x} - e^{x} - e^{-x} + C$

47.) $\frac{d}{dx}\left(y\ln x + y^2 = 0\right)$

$$y\cdot\frac{1}{x} + \ln x\cdot\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

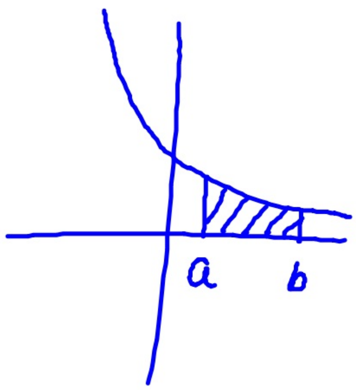
$$\ln x\,\frac{dy}{dx} + 2y\frac{dy}{dx} = -\frac{y}{x}$$

$$\frac{dy}{dx}\left(\ln x + 2y\right) = -\frac{y}{x}$$

$$\frac{dy}{dx} = \frac{-\frac{y}{x}\cdot x}{x\cdot\ln x + 2y\cdot x}$$

$$= \frac{-y}{x\ln x + 2xy}$$

134.) $y=e^{-x}$, $y=0$, $x=a$, $x=b$



$$\int_a^b e^{-x}\,dx = -e^{-x}\Big|_a^b$$

$u=-x$

$du=-1dx$

$$-\left(e^{-b}-e^{-a}\right)$$

$$e^{-a}-e^{-b}$$

55.) $\displaystyle\int_1^3 \frac{e^x}{e^x-1}\,dx$

$u = e^x - 1$
$du = e^x\,dx$

$\ln|e^x-1|\Big|_1^3$

$\ln(x^2+1)$

$\ln|1-x^2|$

$\ln|e^3-1| - \ln|e^1-1|$

$\ln\frac{e^3-1}{e-1} = \ln\frac{\cancel{(e-1)}(e^2+e+1)}{\cancel{e-1}}$

55.) $y = \log_5 \sqrt{x^2 - 1}$

$$y = \frac{1}{2} \log_5 (x^2 - 1)$$

$$y' = \frac{1}{\cancel{2}} \cdot \frac{1}{\ln 5} \cdot \frac{\cancel{2}x}{x^2 - 1}$$

$$y' = \frac{x}{\ln 5 (x^2 - 1)}$$