5.7 Inverse Trigonometric Functions: Integration

- Integrate functions whose antiderivatives involve inverse trigonometric functions.
- Use the method of completing the square to integrate a function.
- Review the basic integration rules involving elementary functions.

Test next Wednesday

5.3: Inverses

5.6: Deriv. inv trig

5.7: anti-deriv inverse trig

$$\int \frac{x}{\sqrt{1-6x^2}}\,dx = -\frac{1}{12}\int u^{-1/2}\,du$$

$u = 1-6x^2$

$du = -12x\,dx$

$$-\frac{1}{12}\cdot\frac{u^{1/2}}{1/2} + C$$

$$-\frac{1}{6}\sqrt{1-6x^2} + C$$

$$\int \frac{x}{(1+x^2)^2}\,dx = \frac{1}{2}\int \frac{1}{u^2}\,du$$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \frac{x}{u} \cdot \frac{du}{2x}$$

$$= \frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|1+x^2| + C$$

$$\int \frac{x+1}{x}\,dx = \int \left(\frac{x}{x} + \frac{1}{x}\right) dx$$

$$\int \left(1 + \frac{1}{x}\right) dx = x + \ln|x| + C$$

#1

$$\int \frac{x+1}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx$$

u-sub

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int u^{-1/2}\,du$

$-\frac{1}{2}\cdot\frac{u^{1/2}}{1/2}$

$-\sqrt{1-x^2}$

arcsin

$a = 1$

$u = x$

$du = 1\,dx$

$+\arcsin x + C$

#2

$$\int \frac{1+x}{4+x^2}\,dx = \int \frac{1}{4+x^2}\,dx + \int \frac{x}{4+x^2}\,dx$$

arctan

$u = x$

$du = 1\,dx$

$a = 2$

u-sub

$u = 4+x^2$

$du = 2x\,dx$

$$\frac{1}{2}\arctan\frac{x}{2} + \frac{1}{2}\ln|4+x^2| + C$$

$$\frac{1}{2}\int \frac{2\cdot x}{1+x^4}\,dx$$

$u = x^2$

$du = 2x\,dx$

$a = 1$

$$\frac{1}{2}\cdot \arctan x^2 + C$$

3.) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$u = e^x$

$du = e^x\,dx$

$a = 1$

$\arcsin\frac{e^x}{1} + C$

~~$u = 1 - e^{2x}$~~

~~$du = -2e^{2x}\,dx$~~

p. 387:

21, 22, 29, 31

33, 35