5.7 Inverse Trigonometric Functions: Integration

- Integrate functions whose antiderivatives involve inverse trigonometric functions.
- Use the method of completing the square to integrate a function.
- Review the basic integration rules involving elementary functions.

These are the last techniques of integration. Get ready to think! You need to consider all of the methods when deciding how to find an integral.

Power rule

U-substitution

ln (u'/u)

long division (degree num > or = degree den.)

inverse trig

#1 $$\int \frac{dx}{\sqrt{4-x^2}} = \arcsin\frac{x}{2} + C$$

$a = 2$

$u = x$

$du = 1\,dx$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

check:

$$\frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} = \frac{\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}} = \frac{\frac{1}{2}}{\frac{\sqrt{4-x^2}}{2}} = \frac{1}{\sqrt{4-x^2}} \checkmark$$

#2 $\int \frac{dx}{\sqrt{1-4x^2}} = \frac{1}{2}\arcsin\frac{2x}{1} + C$

$a = 1$

$u = 2x$

$du = 2dx$

$\int \frac{x}{\sqrt{1-4x^2}}dx = -\frac{1}{8}\int u^{-1/2} du$

$u = 1-4x^2$

$du = -8x\,dx$

3.) $\displaystyle\int \frac{\sin x}{\sqrt{4-\cos^2 x}}\,dx = -\arcsin\frac{\cos x}{2} + C$

$a = 2$

$u = \cos x$

$du = -\sin x\,dx$

#4

$$\int_{\sqrt{3}}^{3} \frac{6}{9+x^2}\,dx = 6\left(\frac{1}{3}\arctan\frac{x}{3}\right)\Big]_{\sqrt{3}}^{3}$$

$a = 3$
$u = x$
$du = 1\,dx$

$$= 2\left(\arctan 1 - \arctan\frac{\sqrt{3}}{3}\right)$$

$$= 2\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = 2\left(\frac{\pi}{12}\right)$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$= \pi/6$$

5) $$\int \frac{6x}{9+x^2}\,dx$$

$$6\int \frac{x}{9+x^2}\,dx = \frac{6}{2}\int \frac{1}{u}\,du$$

$u = 9 + x^2$

$du = 2x\,dx$

$3\ln|u| + C$

$3\ln|9+x^2| + C$

#6

$$\int \frac{t}{t^4 + 16}\,dt = \frac{1}{2}\left(\frac{1}{4}\arctan\frac{t^2}{4}\right) + C$$

$a = 4$

$u = t^2$

$du = 2t\,dt$

$$\frac{1}{8}\arctan\frac{t^2}{4} + C$$

#7 $\displaystyle\int \frac{1\cdot x}{x\,x\sqrt{x^4-4}}\,dx = \frac{1}{2}\left(\frac{1}{2}\operatorname{arcsec}\frac{|x^2|}{2}\right) + $

$a = 2$

$u = x^2$

$du = 2x\,dx$

$$= \frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

8.) $\displaystyle\int_1^4 \frac{4\cdot 1}{4x\sqrt{16x^2-9}}\,dx = \frac{1}{3}\operatorname{arcsec}\frac{|4x|}{3}\Bigg]_1^4$

$\displaystyle\frac{1}{3}\left(\operatorname{arcsec}\frac{16}{3}-\operatorname{arcsec}\frac{4}{3}\right)$

$a = 3$

$u = 4x$

$du = 4\,dx$

9.) $\int \frac{e^x}{\sqrt{25-e^{2x}}}dx = \arcsin\frac{e^x}{5} + C$

$a = 5$

$u = e^x$

$du = e^x dx$

10.) $\displaystyle\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \int (\arcsin x)\left(\frac{1}{\sqrt{1-x^2}}\right.$

$u = \arcsin x$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx$

$\displaystyle\int u^1\,du = \frac{1}{2}u^2 + C$

$\dfrac{1}{2}(\arcsin x)^2 + C$

THEOREM 5.17 INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

See page 385 for all the
integration formulas

Don't get fooled! Analyze the integral and determine what method (if any) must be used.

a. $\displaystyle\int \frac{dx}{x\sqrt{x^2-1}}$ arcsec

b. $\displaystyle\int \frac{x\,dx}{\sqrt{x^2-1}}$ u-sub

c. $\displaystyle\int \frac{dx}{\sqrt{x^2-1}}$

(a) $\displaystyle\int \frac{1}{1+x^4}\,dx$

(b) $\displaystyle\int \frac{x}{1+x^4}\,dx$ arctan

(c) $\displaystyle\int \frac{x^3}{1+x^4}\,dx$ u-sub

$\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ arcsin

(b) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ u-sub

(c) $\displaystyle\int \frac{1}{x\sqrt{1-x^2}}\,dx$