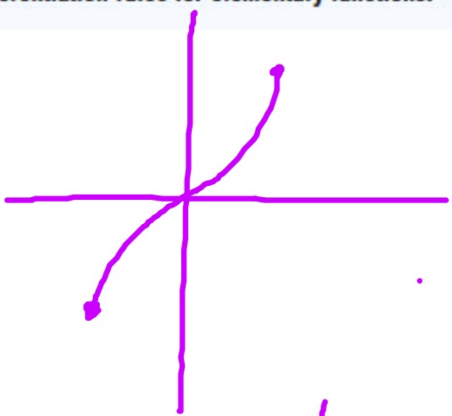
5.6 Inverse Trigonometric Functions: Differentiation

- Develop properties of the six inverse trigonometric functions.
- Differentiate an inverse trigonometric function.
- Review the basic differentiation rules for elementary functions.

$y = \sin x$

$D: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$R: [-1, 1]$

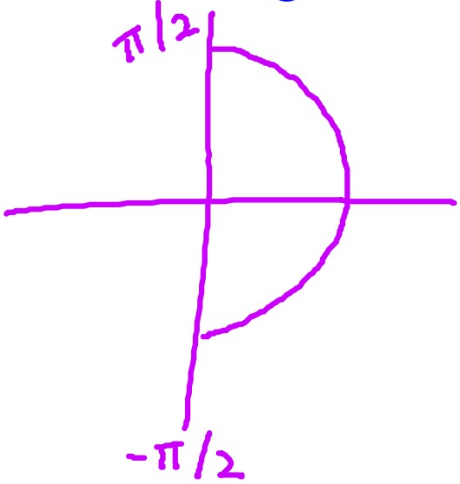


$y = \sin^{-1} x$

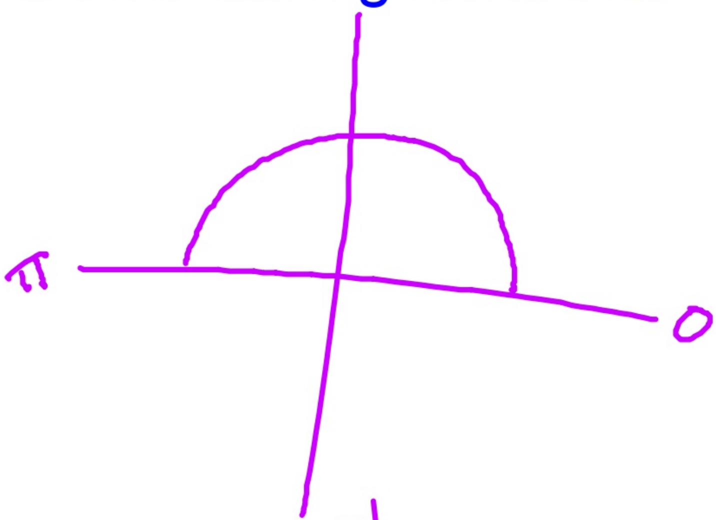
$D: [-1, 1]$

$R: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

The ranges of all 6 inverse trig functions



$\tan^{-1} x$
$\sin^{-1} x$
$\csc^{-1} x$



$\cot^{-1} x$
$\cos^{-1} x$
$\sec^{-1} x$

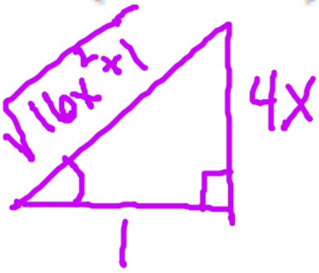
1. $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
2. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$
3. $\arctan(1) = \frac{\pi}{4}$
4. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$
5. $\csc^{-1}(-2) = -\frac{\pi}{6}$
6. $\cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$
7. $\sec^{-1}(\sqrt{2}) = \frac{\pi}{4}$
8. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\tan\left(\arctan(2x-5)\right) = \tan\left(\frac{\pi}{4}\right)$

$2x-5=1$

$x=3$

$\sec(\arctan 4x)$



$(4x)^2+1^2=c^2$

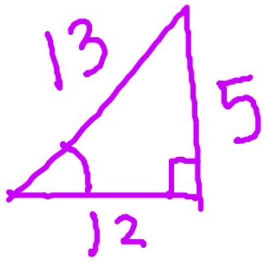
$16x^2+1=c^2$

$\sqrt{16x^2+1}=c$

$\boxed{\frac{\sqrt{16x^2+1}}{1}}$

$\frac{4x}{1}$

$\cos\left(\arcsin\frac{5}{13}\right)$



$\frac{12}{13}$

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37.) $\sin\left(\arcsin(3x - \pi)\right) = \sin\left(\frac{1}{2}\right)$

$$3x - \pi = \sin\left(\tfrac{1}{2}\right)$$

$$x = \frac{\sin\left(\frac{1}{2}\right) + \pi}{3}$$

39.) $\sin(\arcsin\sqrt{2x}) = \sin(\arccos\sqrt{x})$

$$\sqrt{2x} = \sin(\arccos\sqrt{x})$$

$$\sqrt{2x} = \sqrt{1-x}$$

$$2x = 1-x$$

$$3x = 1$$

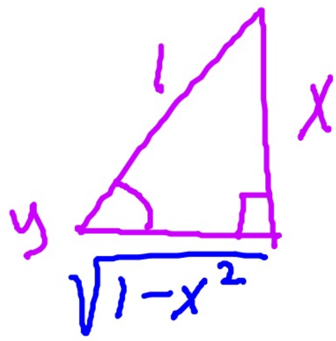
$$x = \frac{1}{3}$$

$$1^2 = \sqrt{x}^2 + b^2$$

$$1-x = b^2$$

$$\sqrt{1-x} = b$$

Find y' for y = arcsin x



$$\sin(y) = \sin(\arcsin x)$$

$$\frac{d}{dx}\left(\sin y = x\right)$$

$$\cos y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{y' = \frac{1}{\sqrt{1-x^2}}}$$

$y = \arcsin u$

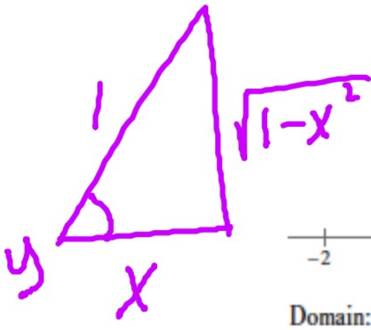
$$y' = \frac{u'}{\sqrt{1-u^2}}$$

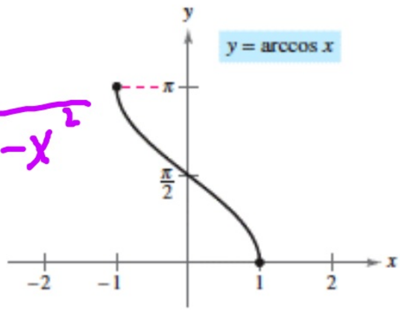
$$= \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$y = \arcsin(x^4)$

$$y' = \frac{4x^3}{\sqrt{1-x^8}}$$

Find y' for y = arccos x





Domain: [−1, 1]
Range : [0, π]

$$\left(\cos y = x\right)\frac{d}{dx}$$

$$-\sin y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{-\sin y}$$

$$= \frac{1}{\frac{-\sqrt{1-x^2}}{1}} = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \arccos u$$

$$y' = \frac{-u'}{\sqrt{1-u^2}}$$

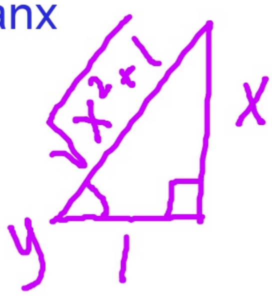
$$y = \arccos\left(\frac{x}{2}\right)$$

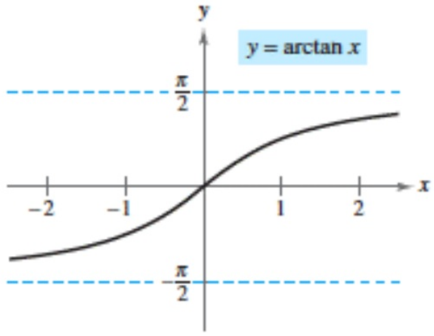
$$y' = \frac{-\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}$$

$$= \frac{-\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}} = \frac{-\frac{1}{2}}{\frac{\sqrt{4-x^2}}{2}}$$

$$= \frac{-1}{\sqrt{4-x^2}}$$

Find y' for y = arctanx





Domain: $(-\infty, \infty)$
Range : $(-\pi/2, \pi/2)$

$$\left(\tan y = x\right) \frac{d}{dx}$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \frac{1}{\left(\sqrt{x^2+1}\right)^2} = \frac{1}{x^2+1}$$

$$y = \arctan\sqrt{x+1} \qquad \frac{u'}{1+u^2}$$

$$y' = \frac{\frac{1}{2}(x+1)^{-1/2}\cdot 1}{1+\left(\sqrt{x+1}\right)^2}$$

$$= \frac{\frac{1}{2\sqrt{x+1}}}{1+(x+1)} = \frac{1}{2\sqrt{x+1}}\cdot\frac{1}{x+2} = \frac{1}{2\sqrt{x+1}\,(x+2)}$$

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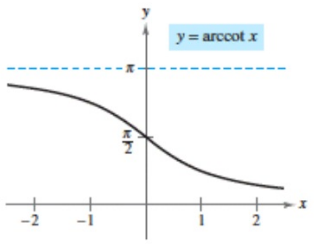
THEOREM 5.16 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

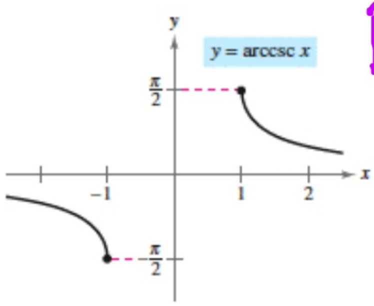
$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$



Domain: $(-\infty, \infty)$
Range: $(0, \pi)$



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Find the equation of the tangent line.

. $y = \arctan \frac{x}{2}$, $\left(2, \frac{\pi}{4}\right)$

Find f'(x)

$f(x) = \operatorname{arcsec} 2x$