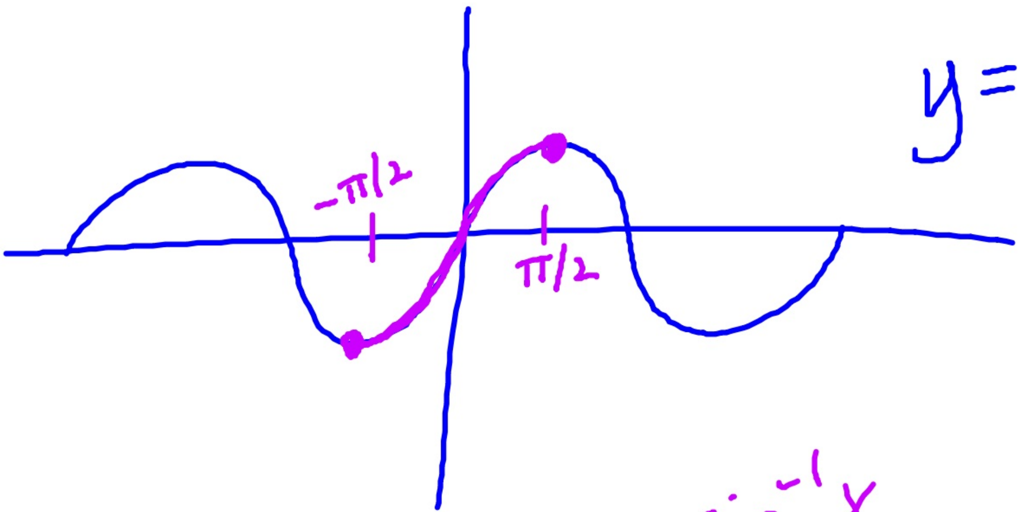
5.6 Inverse Trigonometric Functions: Differentiation

- Develop properties of the six inverse trigonometric functions.
- Differentiate an inverse trigonometric function.
- Review the basic differentiation rules for elementary functions.

DEFINITIONS OF INVERSE TRIGONOMETRIC FUNCTIONS

Function	*Domain*	*Range*
$y = \arcsin x$ iff $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ iff $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ iff $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \operatorname{arccot} x$ iff $\cot y = x$	$-\infty < x < \infty$	$0 < y < \pi$
$y = \operatorname{arcsec} x$ iff $\sec y = x$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi,\quad y \ne \frac{\pi}{2}$
$y = \operatorname{arccsc} x$ iff $\csc y = x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2},\quad y \ne 0$

$y = \sin x$



$y = \sin x$

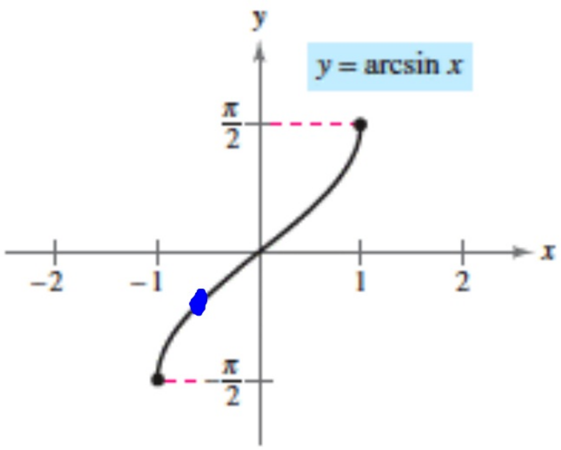
D: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

R: $[-1, 1]$

$y = \sin^{-1} x$

D: $[-1, 1]$

R: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Domain: $[-1, 1]$
Range: $[-\pi/2, \pi/2]$

$$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

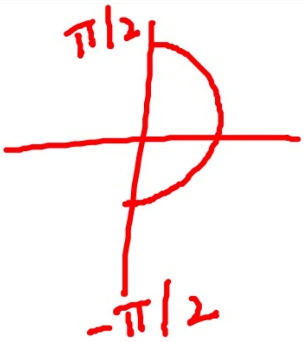
$$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$\sin^{-1}x = \arcsin x$$

arcsinx

arccscx

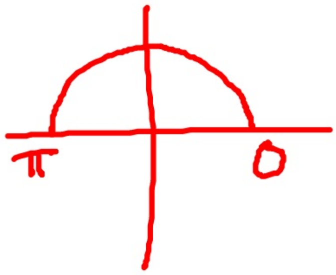
arctanx



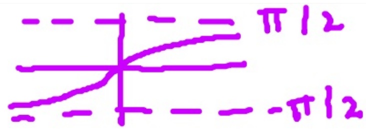
arccosx

arcsecx

arccotx



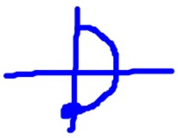
① $\arctan(-1) = -\pi/4$



② $\arccos\left(-\frac{1}{2}\right) = 2\pi/3$

③ $\operatorname{arcsec}(\sqrt{2}) = \pi/4$

④ $\operatorname{arccsc}(-1) = -\pi/2$



⑤ $\operatorname{arccot}(\sqrt{3}) = \pi/6$

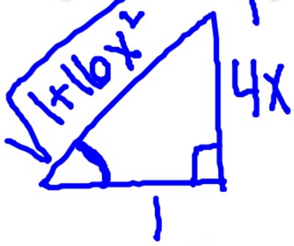
$\tan\left(\arctan(2x - 5)\right) = \tan\left(\frac{\pi}{4}\right)$

$2x - 5 = 1$

$x = 3$

$f(f^{-1}(x)) = x$

$\sec(\arctan \frac{4x}{1})$

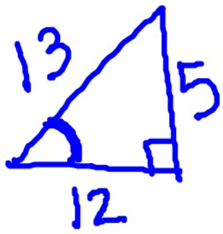


$1^2 + (4x)^2 = c^2$

$\frac{\sqrt{1+16x^2}}{1}$

$\sqrt{1+16x^2}$

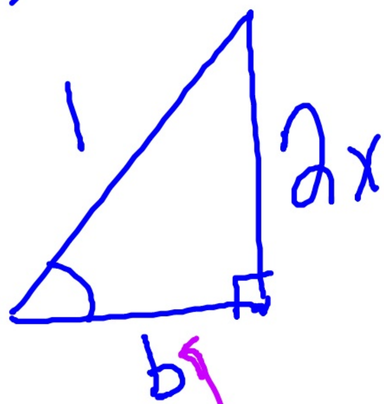
$\cos\left(\arcsin \frac{5}{13}\right)$



$\frac{12}{13}$

27.) $\cos(\arcsin 2x)$

$$\frac{\sqrt{1-4x^2}}{1}$$



$b^2 + (2x)^2 = 1^2$

$b = \sqrt{1-4x^2}$

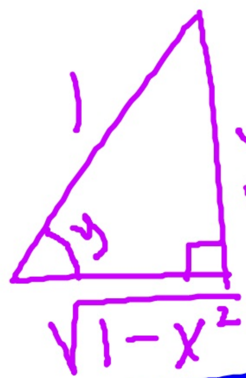
Find y' for y = arcsin x

$$\sin(y) = \sin(\arcsin x)$$

$$(\sin y = x)\frac{d}{dx}$$

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$



$$y = \arcsin u$$

$$y' = \frac{u'}{\sqrt{1-u^2}}$$

$$g(x) = \arcsin(x^3)$$

$$g'(x) = \frac{3x^2}{\sqrt{1-x^6}}$$

$$f(x) = \arcsin \frac{x}{2}$$

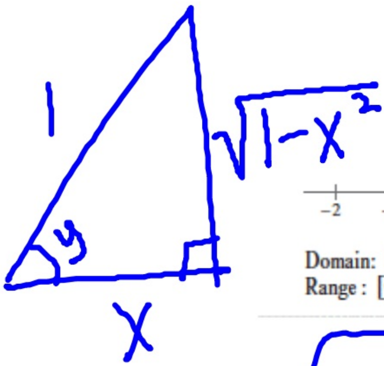
$$f'(x) = \frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}$$

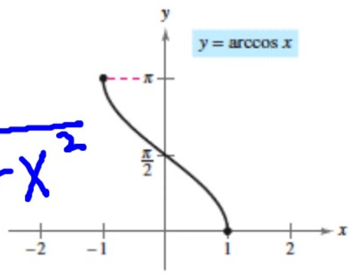
$$= \frac{\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}} = \frac{\frac{1}{2}}{\frac{\sqrt{4-x^2}}{2}}$$

$$\frac{1}{2} \div \frac{\sqrt{4-x^2}}{2} = \frac{1}{\sqrt{4-x^2}}$$

Find y' for y = arccos x

$$\frac{d}{dx}\left(\cos y = x\right)$$





Domain: $[-1, 1]$
Range : $[0, \pi]$

$$-\sin y \frac{dy}{dx} = 1$$

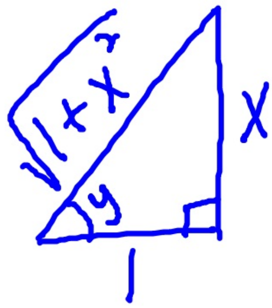
$$\frac{dy}{dx} = \frac{1}{-\sin y} = \frac{1}{-\sqrt{1-x^2}}$$

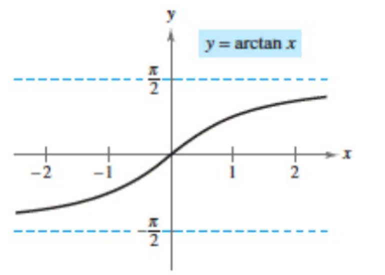
$$y = \arccos u$$

$$y' = \frac{-u'}{\sqrt{1-u^2}}$$

Find y' for y = arctanx

$$\left(\tan y = x\right)\frac{d}{dx}$$





Domain: $(-\infty, \infty)$
Range: $(-\pi/2, \pi/2)$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \left(\frac{1}{\sqrt{1+x^2}}\right)^2$$

$$= \frac{1}{1+x^2}$$

$$y = \arctan u$$

$$y' = \frac{u'}{1+u^2}$$

THEOREM 5.16 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

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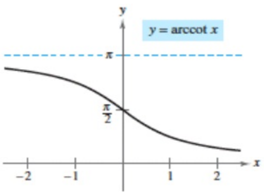
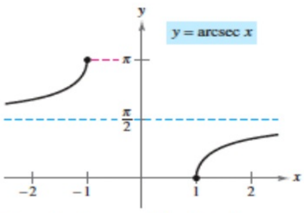
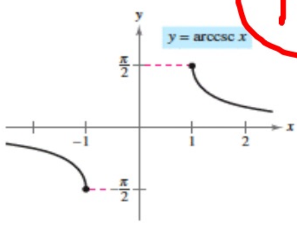


Domain: $(-\infty, \infty)$
Range: $(0, \pi)$



Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[0, \pi/2) \cup (\pi/2, \pi]$



Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[-\pi/2, 0) \cup (0, \pi/2]$
Figure 5.29

Find the equation of the tangent line.

$$y=\arctan\frac{x}{4} \quad \left(4, \frac{\pi}{4}\right)$$

$$y' = \frac{\frac{1}{4}\cdot 16}{16\cdot 1+\frac{x^2}{16}\cdot 16} = \frac{4}{16+x^2}$$

$$y'(4) = \frac{4}{16+16} = \frac{1}{8}$$

$$\boxed{y-\frac{\pi}{4}=\frac{1}{8}(x-4)}$$

Find f'(x)

$$f(x) = \operatorname{arcsec} 2x$$

$$f'(x) = \frac{2}{|2x|\sqrt{4x^2-1}}$$

$$= \frac{1}{|x|\sqrt{4x^2-1}}$$

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43-48 All
63, 65