75.) $y = \tan x$

$$\int_0^{\pi/4} \tan x \, dx = -\ln|\cos x| \Big|_0^{\pi/4}$$

$$-\left(\ln\left|\frac{\sqrt{2}}{2}\right| - \cancel{\ln|1|}\right)$$

$$\boxed{-\ln\frac{\sqrt{2}}{2}} = \ln\frac{2}{\sqrt{2}}$$

35.) $\int \cos 3\theta \, d\theta - \int 1 \, d\theta$

$\frac{1}{3} \sin 3\theta - \theta + C$

113.) $\int \frac{5-e^{x}}{e^{2x}}\,dx = \int \frac{5}{e^{2x}}\,dx - \int \frac{e^{x}}{e^{2x}}\,dx$

$\int 5e^{-2x}\,dx - \int e^{-x}\,dx$

$u = -2x$
$du = -2\,dx$

$u = -x$
$du = -1\,dx$

$\int 5e^{-2x}\,dx$

$\int 5e^{u}\cdot\frac{du}{-2}$

$-\frac{5}{2}\int e^{u}\,du + \int e^{u}\,du$

$-\frac{5}{2}e^{-2x} + e^{-x} + C$

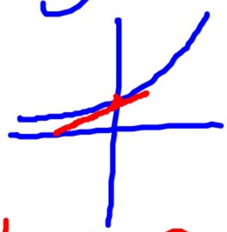
133.) $y=e^x,\ x=0,\ x=5,\ y=0$

$$\int_0^5 e^x\,dx = e^x \Big|_0^5$$

$$= e^5 - 1$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$y = 2^x$

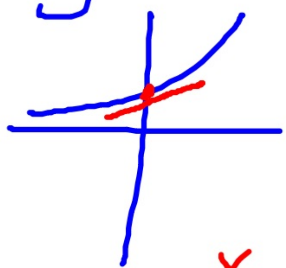


$y' = \ln 2 \cdot 2^x$

$y'(0) = \ln 2$

$.693$

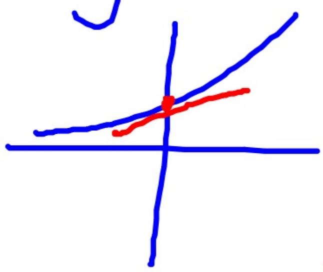
$y = e^x$



$y' = e^x$

$y'(0) = \underline{\underline{\underline{1}}}$

$y = 3^x$



$y' = \ln 3 \cdot 3^x$

$y'(0) = \ln 3$

1.0986

$$\int e^{7x}\,dx = \frac{1}{7}e^{7x} + C$$

$$\int \frac{1+e^x}{x+e^x}\,dx = \ln|x+e^x| + C$$

109) $\int e^x\sqrt{1-e^x}\,dx$

$u = 1-e^x$

$du = -e^x dx$

$\frac{du}{-e^x} = dx$

$\int e^x\sqrt{u}\cdot\frac{du}{-e^x}$

$-\int u^{1/2}du = -\frac{2}{3}u^{3/2}$

$-\frac{2}{3}(1-e^x)^{3/2} + C$

107.) $\int \frac{e^{-x}}{1+e^{-x}} dx$

$u = 1+e^{-x}$
$du = -e^{-x} dx$

$-\int \frac{1}{u} du$

$-\ln|u| + C$

$-\ln|1+e^{-x}| + C$

121) $\int_{1}^{3} e^{3/x} \cdot \frac{1}{x^2}\, dx$

$u = \frac{3}{x} = 3x^{-1}$

$du = -3x^{-2} dx$

$du = \frac{-3}{x^2} dx$

$\frac{x^2 du}{-3} = dx$

$-\frac{1}{3} \int_{3}^{1} e^{u} du$

$-\frac{1}{3} e^{3/x} \Big|_{1}^{3} = -\frac{1}{3}\left(e^{1} - e^{3}\right)$

59.) $\int_1^2 \frac{1-\cos\theta}{\theta - \sin\theta}\,d\theta$

$u = \theta - \sin\theta$

$du = 1-\cos\theta\,d\theta$

$\int \frac{1}{u}\,du$

$\ln|u| = \ln|\theta - \sin\theta|\Big|_1^2$

$\ln|2-\sin 2| - \ln|1-\sin 1|$

$\ln\left|\frac{2-\sin 2}{1-\sin}\right|$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\cot x + \csc x| + C$$

5.5 Bases Other Than *e* and Applications

- Define exponential functions that have bases other than *e*.
- Differentiate and integrate exponential functions that have bases other than *e*.
- Use exponential functions to model compound interest and exponential growth.

$$\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + C$$

$y = a^x$

$y' = \ln a \cdot a^x$

#1

$$\int_0^3 2^x\,dx$$

$$\frac{1}{\ln 2}\cdot 2^x \Big|_0^3$$

$$\frac{1}{\ln 2}(8-1)$$

$$\frac{7}{\ln 2}$$

#2

$$\int_{-2}^{2} 4^{x/2}\,dx$$

$u=\frac{x}{2}$

$du=\frac{1}{2}dx$

$$2\int 4^u\,du$$

$$2\cdot\frac{1}{\ln 4}\cdot 4^u=\frac{2}{\ln 4}\cdot 4^{x/2}\Big|_{-2}^{2}$$

$$\frac{2}{\ln 4}\left(4^1-\frac{1}{4}\right)=\frac{2}{\ln 4}\left(\frac{15}{4}\right)$$

$$\frac{30}{4\ln 4}=\frac{15}{2\ln 4}=\frac{15}{\ln 16}$$

#3 $\int x \cdot 6^{x^2} dx$ $\quad u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int 6^u du$

$= \frac{1}{2}\frac{1}{\ln 6} \cdot 6^u + C = \frac{1}{2\ln 6} \cdot 6^{x^2} + C$

4) find the area enclosed by

$y = 2^{-x}$, $x = 0$, $x = 3$, $y = 0$