## 5.4 Exponential Functions: Differentiation and Integration

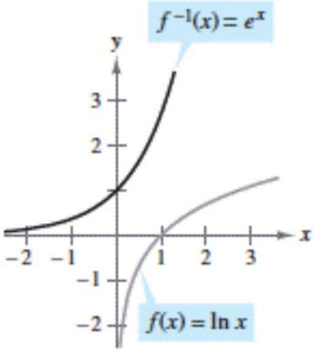
**DEFINITION OF THE NATURAL EXPONENTIAL FUNCTION**

The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

$$y = e^x \qquad \text{if and only if} \qquad x = \ln y.$$




The inverse function of the natural logarithmic function is the natural exponential function.

**THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS**

Let $u$ be a differentiable function of $x$.

1. $\int e^x \, dx = e^x + C$    2. $\int e^u \, du = e^u + C$

$y = e^x$

$y' = e^x$

$y = e^{2x}$

$y' = 2e^{2x}$

#2

$$\int \frac{e^{2x}}{1 + e^{2x}}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$$

$u = 1 + e^{2x}$

$du = 2e^{2x}\,dx$

$$= \frac{1}{2}\ln|1 + e^{2x}| + C$$

#1 $\int e^{1-3x}\,dx = -\frac{1}{3}\int e^{u}\,du$

$u = 1-3x$

$du = -3dx$

$\frac{du}{-3} = dx$

$-\frac{1}{3}e^{u} + C$

$-\frac{1}{3}e^{1-3x} + C$

Check: $-\frac{1}{3}\cdot e^{1-3x}(-3) = e^{1-3x}$ ✓

#3 $\int_0^1 \frac{e^x}{5-e^x}\,dx$

$u = 5 - e^x$

$du = -e^x dx$

$-\int \frac{1}{u}\,du$

$-\ln|u|$

$= -\ln|5-e^x|\Big]_0^1$

$= -\left(\ln|5-e|-\ln 4\right)$

$-\ln|5-e|+\ln 4$

$\ln 4 - \ln|5-e|$

$\ln \frac{4}{5-e}$

#4 $\displaystyle\int_1^2 \frac{e^{1/x^2}}{x^3}\,dx = \int e^{1/x^2}\cdot\frac{1}{x^3}\,dx$

$u = \frac{1}{x^2} = x^{-2}$

$du = -2x^{-3}dx$

$du = -\frac{2}{x^3}dx$

$-\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^{1/x^2}\Big]_1^2$

$-\frac{1}{2}\left(e^{1/4} - e^1\right)$

#5 $$\int \frac{e^{3x}+2e^{x}+1}{e^{x}}\,dx = \int\left(e^{2x}+2+e^{-x}\right)dx$$

$$\int e^{2x}dx + \int 2dx + \int e^{-x}dx$$

$u=2x$

$du=2dx$

$u=-x$

$du=-1dx$

$$\frac{1}{2}\int e^{u}$$

$$-\int e^{u}du$$

$$\frac{1}{2}e^{2x}+2x-e^{-x}+C$$

37.) $y = e^{3x}$ $(0,1)$

$y' = 3e^{3x}$

$y'(0) = 3$

$y - 1 = 3(x - 0)$ or

$y = 3x + 1$

49.) $g(t) = \left(e^{-t} + e^{t}\right)^3$

$g'(t) = 3\left(e^{-t} + e^{t}\right)^2 \cdot \left(-e^{-t} + e^{t}\right)$

71.) $xe^y + ye^x = 1 \qquad (0,1)$

$$xe^y \frac{dy}{dx} + e^y \cdot 1 + ye^x + e^x \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-e^y - ye^x}{xe^y + e^x}$$

$$\boxed{y-1 = (-e-1)(x-0)}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{-e^1 - 1 \cdot e^0}{0e^1 + e^0} = \frac{-e^1 - 1}{1}$$

$$= (-e^1 - 1)$$

61.) $f(x) = e^{1-x}$ $(1, 1)$

$f'(x) = e^{1-x} \cdot (-1)$

$f'(1) = e^{0} \cdot (-1)$

$= -1$

$y - 1 = -1(x - 1)$

83.) $f(x) = x^2 e^{-x}$

$$f'(x) = x^2(-e^{-x}) + e^{-x} \cdot 2x$$

$$0 = -e^{-x} \cdot x(x-2)$$

$$x = 0, 2$$

$f'$: $-$ $+$ $-$

$-10$ $0$ $1$ $2$

rel. max $\left(2, \frac{4}{e^2}\right)$

rel. min $(0, 0)$

$y = x \ln x$  incr. ?

$y' = x \frac{1}{x} + \ln x \cdot 1$

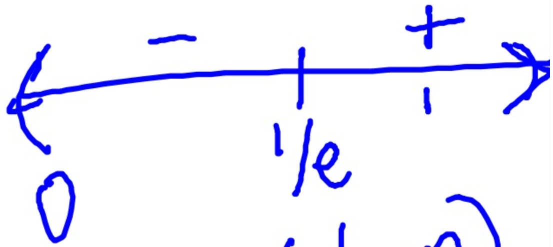
$y' = 1 + \ln x$

$0 = 1 + \ln x$

$e^{-1} = e^{\ln x}$

$e^{-1} = x$

$\frac{1}{e} = x$




$\left(\frac{1}{e}, \infty\right)$