5.4 Exponential Functions: Differentiation and Integration

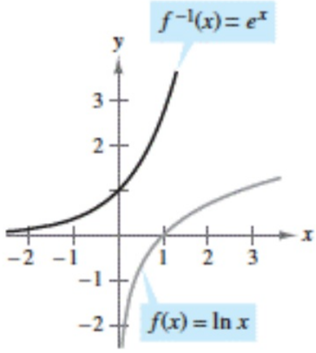
DEFINITION OF THE NATURAL EXPONENTIAL FUNCTION

The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

$$y = e^x \qquad \text{if and only if} \qquad x = \ln y.$$



The inverse function of the natural logarithmic function is the natural exponential function.

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$

THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION

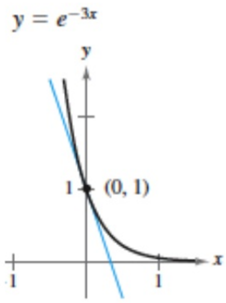
Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$
2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

$y = e^{4x}$

$y' = e^{4x} \cdot 4$

#1: Find the equation of the tangent line at the given point



$y = e^{-3x}$

$(0, 1)$

$y' = e^{-3x} \cdot -3$

$y'(0) = -3$

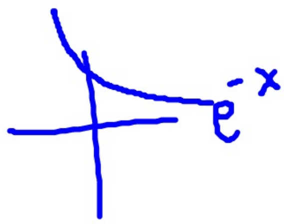
$y - 1 = -3(x - 0)$

#2: Find y'

$y = x^2 e^{-x}$

$y' = x^2 \cdot e^{-x} \cdot -1 + e^{-x} \cdot 2x$

$y' = -xe^{-x}(x - 2)$



#3: Find y'.

$$y = \frac{e^{2x}}{e^{2x}+1}$$

$$y' = \frac{(e^{2x}+1)2e^{2x} - e^{2x}(2e^{2x})}{(e^{2x}+1)^2}$$

$$y' = \frac{2e^{4x}+2e^{2x}-2e^{4x}}{(e^{2x}+1)^2}$$

$$y' = \frac{2e^{2x}}{(e^{2x}+1)^2}$$

#4: Find dy/dx

$$\frac{d}{dx}\left(e^{xy} + x^2 - y^2 = 10\right)$$

$$e^{xy}\left(x\frac{dy}{dx} + y \cdot 1\right) + 2x - 2y\frac{dy}{dx} = 0$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} + 2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}$$

5.) $y = \ln(1+e^{5x})^3$

$y = 3\ln(1+e^{5x})$

$y' = 3 \cdot \dfrac{5e^{5x}}{1+e^{5x}}$

$= \dfrac{15e^{5x}}{1+e^{5x}}$

6.) $F(x) = \int_0^{e^{2x}} \ln(t+1)\,dt$

$F'(x) = \ln(e^{2x}+1) \cdot 2e^{2x}$

THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x\,dx = e^x + C$ **2.** $\int e^u\,du = e^u + C$

#5 $\int e^{1-3x}\,dx$

#6

$$\int \frac{e^{2x}}{1+e^{2x}}\,dx$$

#7

$$\int_0^1 \frac{e^x}{5 - e^x}\,dx$$