# 5.4 Exponential Functions: Differentiation and Integration

- Develop properties of the natural exponential function.
- Differentiate natural exponential functions.
- Integrate natural exponential functions.

55.) $y = \ln\left(x\sqrt{x^2-1}\right)$

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2-1}$

$= \frac{1}{x} + \frac{x}{x^2-1}$

65) $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

$$f(x) = \frac{1}{2}\ln(4+x^2) - \ln x$$

$$f'(x) = \frac{1}{2}\cdot\frac{2x}{4+x^2} - \frac{1}{x}$$

$$= \frac{x}{4+x^2} - \frac{1}{x}$$

$$y = x \ln x$$

$$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$y' = 1 + \ln x$$

$$y = \frac{x}{\ln x}$$

$$y' = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{(\ln x)^2}$$

$$y' = \frac{\ln x - 1}{(\ln x)^2}$$

$$\ln x^2 \neq (\ln x)^2$$

$$y = x \ln x^3 = 3x \ln x$$

$$y = \frac{x}{\ln x^3} = \frac{x}{3 \ln x}$$

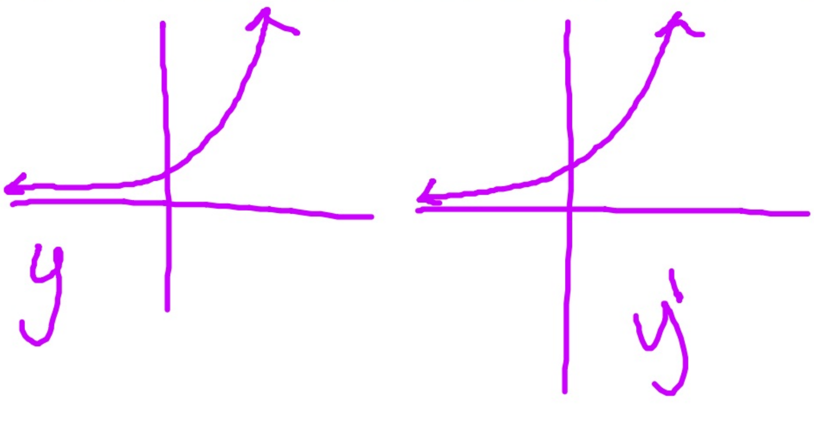
**THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION**

Let $u$ be a differentiable function of $x$.

1. $\dfrac{d}{dx}[e^x] = e^x$

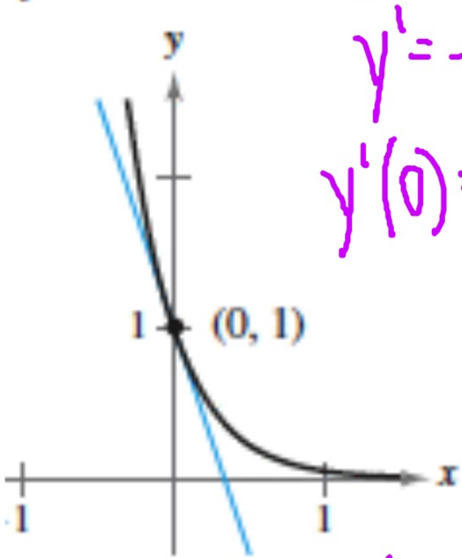
2. $\dfrac{d}{dx}[e^u] = e^u \dfrac{du}{dx}$

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$



#1: Find the equation of the tangent line at the given point

$y = e^{-3x}$




$$y' = e^{-3x}(-3)$$

$$y' = -3e^{-3x}$$

$$y'(0) = -3$$

$$y - 1 = -3(x - 0)$$

#2: Find y'

$$y = x^2 e^{-x}$$

$$y' = x^2 \cdot e^{-x}(-1) + e^{-x} \cdot 2x$$

$$y' = xe^{-x}(-x + 2)$$

or

$$-xe^{-x}(x - 2)$$

#3: Find y'.

$$e^{2x} \cdot e^{2x} = e^{4x}$$

$$y = \frac{e^{2x}}{e^{2x} + 1}$$

$$y' = \frac{(e^{2x}+1)2e^{2x} - e^{2x} \cdot 2e^{2x}}{(e^{2x}+1)^2}$$

$$= \frac{\cancel{2e^{4x}} + 2e^{2x} - \cancel{2e^{4x}}}{(e^{2x}+1)^2} = \boxed{\frac{2e^{2x}}{(e^{2x}+1)^2} = y'}$$

#4 $f(x) = \ln\left(\frac{e^x}{e^x+1}\right)$

$$f(x) = \ln e^x - \ln(e^x+1)$$

$$f(x) = x - \ln(e^x+1)$$

$$f'(x) = 1 - \frac{e^x}{e^x+1}$$

#5 $g(x) = e^{\sin x}$

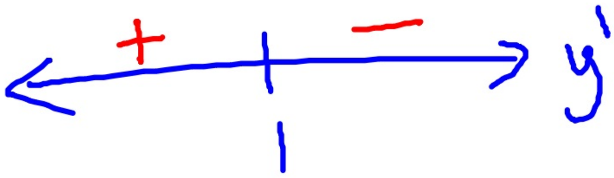
$g'(x) = e^{\sin x} \cdot \cos x$

#6 Find relative extrema for $y = xe^{-x}$

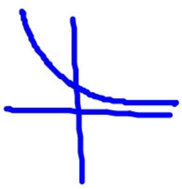
$D: (-\infty, \infty)$

$$y' = x \cdot e^{-x}(-1) + e^{-x} \cdot 1$$

$$= -e^{-x}(x-1)$$




Relative maximum at (1, 1/e) becaue y'
changes from positive to negative
at this point



#7: Find dy/dx

$$\frac{d}{dx}\left(e^x + e^y + y^2 - 2x = 4\right)$$

$$e^x + e^y \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} - 2 = 0$$

$$\frac{dy}{dx}\left(e^y + 2y\right) = 2 - e^x$$

$$\frac{dy}{dx} = \frac{2-e^x}{e^y+2y}$$