

## 5.4

## Exponential Functions: Differentiation and Integration

- Develop properties of the natural exponential function.
- Differentiate natural exponential functions.
- Integrate natural exponential functions.

$$55.) \quad y = \ln(x\sqrt{x^2-1})$$

$$y = \ln x + \frac{1}{2} \ln(x^2 - 1)$$

$$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2 - 1}$$

$$= \frac{1}{x} + \frac{x}{x^2 - 1}$$

$$65) f(x) = \ln \left( \frac{\sqrt{4+x^2}}{x} \right)$$

$$f(x) = \frac{1}{2} \ln(4+x^2) - \ln x$$

$$f'(x) = \frac{1}{2} \cdot \frac{2x}{4+x^2} - \frac{1}{x}$$

$$= \frac{x}{4+x^2} - \frac{1}{x}$$

$$y = x \ln x$$

$$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$y' = 1 + \ln x$$

$$y = \frac{x}{\ln x}$$

$$y' = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{(\ln x)^2}$$

$$y' = \frac{\ln x - 1}{(\ln x)^2}$$

$$\ln x^2 \neq (\ln x)^2$$

$$y = x \ln x^3 = 3x \ln x$$

$$y = \frac{x}{\ln x^3} = \frac{x}{3 \ln x}$$

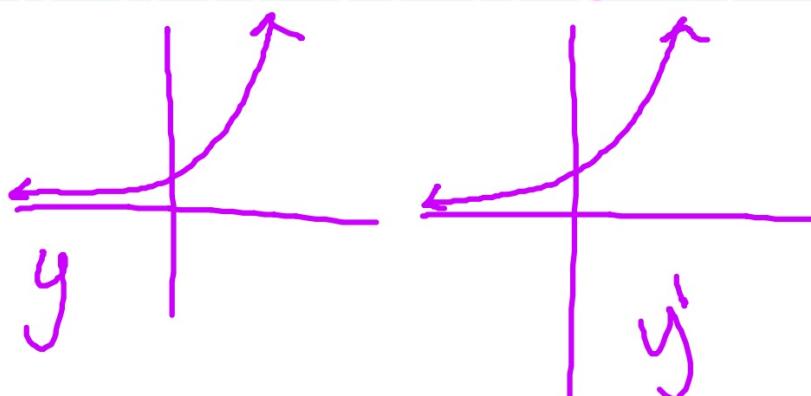
### **THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION**

Let  $u$  be a differentiable function of  $x$ .

$$1. \frac{d}{dx}[e^x] = e^x$$

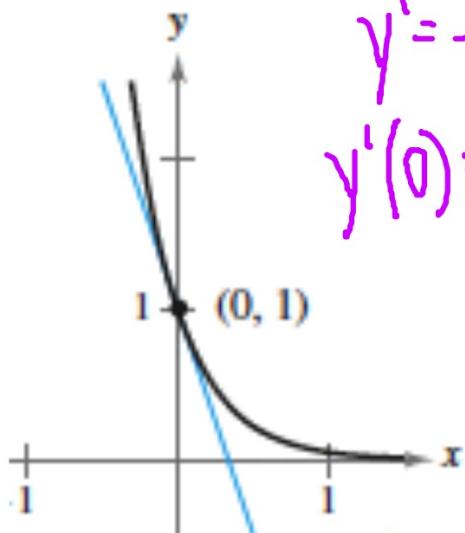
$$2. \frac{d}{dx}[e^u] = e^u \frac{du}{dx}$$

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$



#1: Find the equation of the tangent line at the given point

$$y = e^{-3x}$$



$$y - 1 = -3(x - 0)$$

#2: Find  $y'$

$$y = x^2 e^{-x}$$

$$y' = x^2 \cdot e^{-x}(-1) + e^{-x} \cdot 2x$$

$$y' = x e^{-x}(-x + 2)$$

$$\text{OR}$$

$$-x e^{-x}(x - 2)$$

#3: Find  $y'$ .

$$e^{2x} \cdot e^{2x} = e^{4x}$$

$$y = \frac{e^{2x}}{e^{2x} + 1}$$

$$y' = \frac{(e^{2x} + 1)2e^{2x} - e^{2x} \cdot 2e^{2x}}{(e^{2x} + 1)^2}$$

$$= \frac{2e^{2x} + 2e^{2x} - 2e^{4x}}{(e^{2x} + 1)^2} = \boxed{\frac{2e^{2x}}{(e^{2x} + 1)^2} = y'}$$

$$\#4 \quad f(x) = \ln\left(\frac{e^x}{e^x + 1}\right)$$

$$f(x) = \ln e^x - \ln(e^x + 1)$$

$$f(x) = x - \ln(e^x + 1)$$

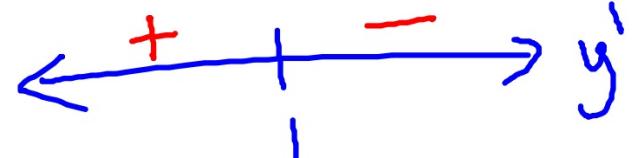
$$f'(x) = 1 - \frac{e^x}{e^x + 1}$$

$$\#5 \quad g(x) = e^{\sin x}$$

$$g'(x) = e^{\sin x} \cdot \cos x$$

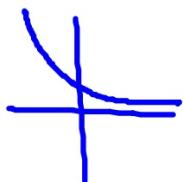
#6 Find relative extrema for  $y = xe^{-x}$

$$D: (-\infty, \infty)$$

$$\begin{aligned}y' &= x \cdot e^{-x}(-1) + e^{-x} \cdot 1 \\&= -e^{-x}(x-1)\end{aligned}$$


A sign chart for the derivative  $y'$ . It consists of a horizontal line with three regions. The first region to the left of the origin is marked with a red '+' above the line. The second region, containing the origin, is marked with a blue '+' above the line. The third region to the right of the origin is marked with a red '-' above the line. A vertical tick mark is placed on the line at  $x=1$ , indicating a change in sign.

Relative maximum at  $(1, 1/e)$  because  $y'$  changes from positive to negative at this point



#7: Find  $dy/dx$

$$\frac{d}{dx} \left( e^x + e^y + y^2 - 2x = 4 \right)$$

$$e^x + e^y \cdot \frac{dy}{dx} + 2y \frac{dy}{dx} - 2 = 0$$

$$\frac{dy}{dx} (e^y + 2y) = 2 - e^x$$

$$\frac{dy}{dx} = \frac{2 - e^x}{e^y + 2y}$$