5.4 Exponential Functions: Differentiation and Integration

- Develop properties of the natural exponential function.
- Differentiate natural exponential functions.
- Integrate natural exponential functions.

THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$
2. $\int e^u du = e^u + C$

#1 $\int e^{1-3x}\,dx$

$u = 1-3x$

$du = -3dx$

$\frac{du}{-3} = dx$

$\int e^u \cdot \frac{du}{-3}$

$-\frac{1}{3}\int e^u du$

$-\frac{1}{3}e^u + C$

$-\frac{1}{3}e^{1-3x} + C$

#2 $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

$u = 1+e^{2x}$

$du = 2e^{2x}dx$

$\frac{du}{2e^{2x}} = dx$

$\int \frac{e^{2x}}{u} \cdot \frac{du}{2e^{2x}}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|1+e^{2x}| + C$

#3 $\int_0^1 \frac{e^x}{5-e^x}\,dx$

$u = 5 - e^x$

$du = -e^x dx$

$-\int \frac{1}{u}\,du = -\ln|5-e^x|\Big|_0^1$

$-\left(\ln(5-e) - \ln(4)\right)$ ✓

$-\ln\frac{5-e}{4}$

$-\ln(5-e) + \ln 4$ ✓

$\ln\frac{4}{5-e}$ ✓

4.) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$\int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx$

$\int e^{u} \cdot \frac{1}{\sqrt{x}} 2\sqrt{x}\,du$

$2\int e^{u}\,du = 2e^{u} + C = 2e^{\sqrt{x}} + C$

5.) $\int \frac{2-e^x}{e^x}\,dx$

$\int \frac{2}{e^x} - \frac{e^x}{e^x}\,dx = \int 2e^{-x}\,dx - \int 1\,dx$

$u = -x$

$du = -1\,dx$

$-2\int e^u\,du - x + C$

$-2e^{-x} - x + C$

6.) $\int e^x \sin(e^x)\,dx$

$u = e^x$

$du = e^x\,dx$

$\frac{du}{e^x} = dx$

$\int e^x \sin u \cdot \frac{du}{e^x}$

$\int \sin u\,du$

$-\cos u + C$

$-\cos(e^x) + C$

7.) $\int x^2 e^{x^3} dx$

$\frac{1}{3} e^{x^3} + C$

Check: $\frac{1}{3} e^{x^3} \cdot 3x^2$

$x^2 e^{x^3}$