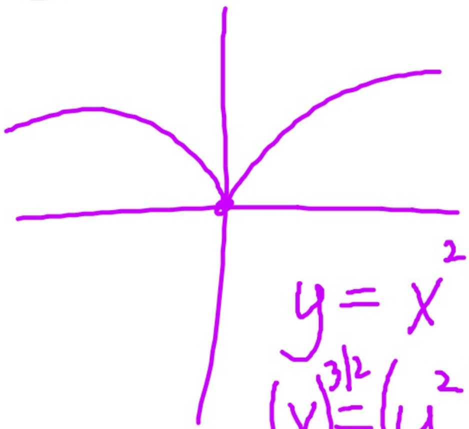
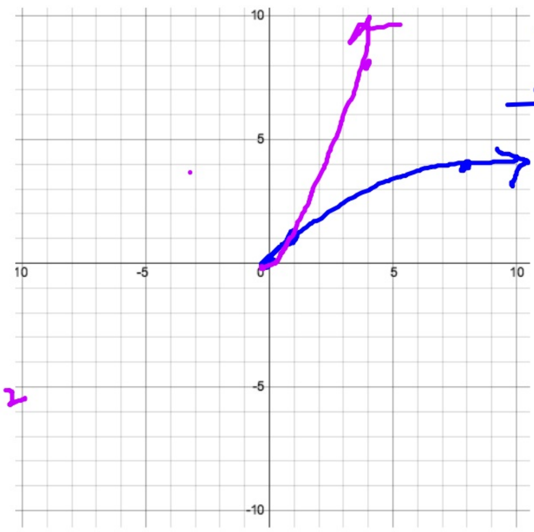
33.) $f(x) = x^{2/3}$





x	$f(x)$
0	0
1	1
8	4

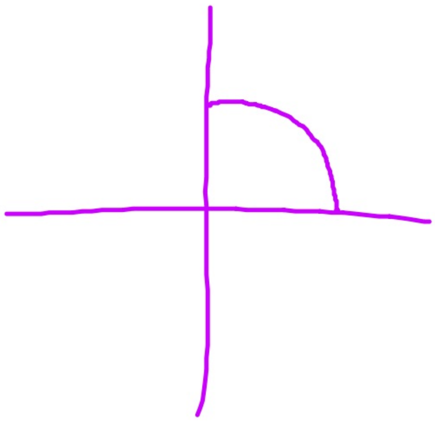
$y = x^{2/3}$

$(x)^{3/2} = (y^{2/3})^{3/2}$

$x^{3/2} = y$

$f^{-1}(x) = x^{3/2}$

29.) $f(x) = \sqrt{4-x^2}$ $[0, 2]$



$x = \sqrt{4-y^2}$

$x^2 = 4-y^2$

$y^2 = 4-x^2$

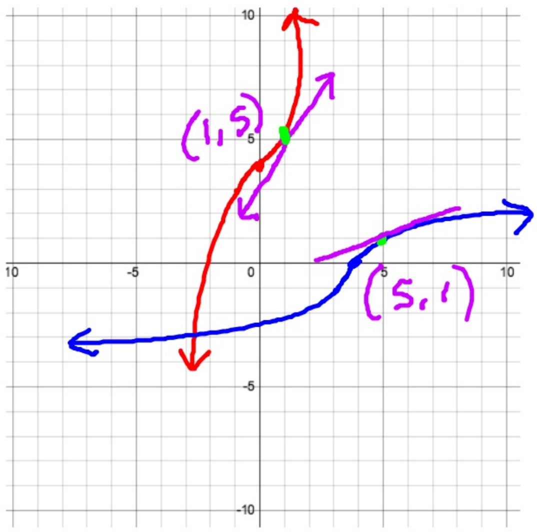
$y = \sqrt{4-x^2}$

5.3 Inverse Functions

$f(x) = x^3 + 4$ $\qquad$ $f^{-1}(x) = \sqrt[3]{x-4}$

$f'(x) = 3x^2$ $\qquad$ $(f^{-1})'(x) = \frac{1}{3}(x-4)^{-2/3}$

$f'(1) = 3$ $\qquad$ $(f^{-1})'(5) = \frac{1}{3}$



1 $f(x) = 2x^3 + 3x$ $(f^{-1})'(5) = \frac{1}{9}$

Verify the function is 1:1 (monotonic: a function that is always increasing or always decreasing)

$f'(x) = 6x^2 + 3$ $\xleftrightarrow{+}$ f'

$0 = 6x^2 + 3$

no critical

f(x) is monotomic because

f'>0

Find the derivative of the inverse at x = 5.

$5 = 2x^3 + 3x$

$1 = x$

$f'(x) = 6x^2 + 3$

$f'(1) = 9 \longrightarrow (f^{-1})'(5) = \frac{1}{9}$

$f: (1, 5)$

$f^{-1}: (5, 1)$

THEOREM 5.9 THE DERIVATIVE OF AN INVERSE FUNCTION

Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0.$$

(2) $f(x) = x^3 - \frac{4}{x}$ D: $(0, \infty)$

f(x) and g(x) are inverses. Find g'(6)

$6 = x^3 - \frac{4}{x}$

$2 = x$

$f'(x) = 3x^2 + \frac{4}{x^2}$

$f'(2) = 12 + 1$

$= 13$

$g'(6) = \frac{1}{13}$

g: (6, 2)

f: (2, 6)

3) Let f be a differentiable function with $f(3) = 15$, $f'(3) = -8$, $f'(6) = -2$, $f(6) = 3$. The function g is differentiable and $g(x) = f^{-1}(x)$. What is the value of $g'(3)$?

a) -1/8

b) -1/2

c) 1/6

d) 1/3

e) not possible

$f : (6, 3)$

$g : (3, 6)$

#4

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$g^{-1}: (2, 1)$

$g: (1, 2)$

$g'(1) = 5$

$(2, 1)$ $m = \frac{1}{5}$

$y - 1 = \frac{1}{5}(x - 2)$