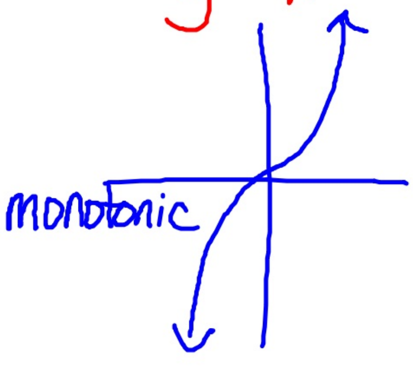
5.3: Inverse Functions

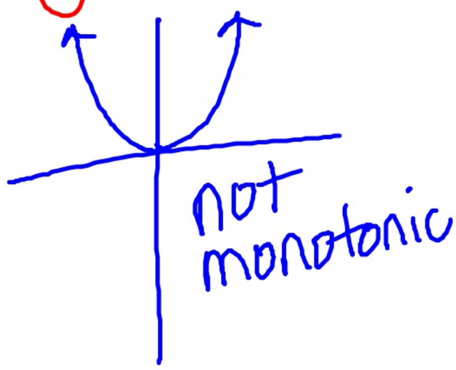
In order for a function to have an inverse function, the function must be monotonic (always increasing or always decreasing)

Synonym for monotonic: 1 to 1

$y = x^3$



$y = x^2$



Proving a function's inverse function

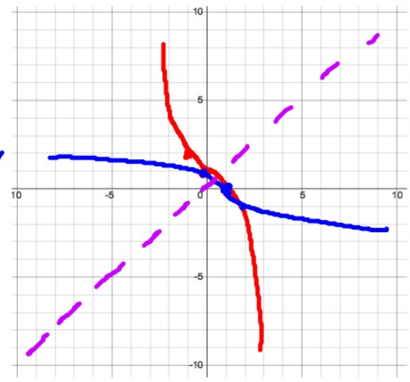
Two functions are inverses if $f(g(x))=x$ and $g(f(x))=x$

① $f(x)=1-x^3$

$g(x)=\sqrt[3]{1-x}$

$f(g(x))=1-\left(\sqrt[3]{1-x}\right)^3=1-(1-x)=x$

$g(f(x))=\sqrt[3]{1-(1-x^3)}=x$



Find the inverse function algebraically and graphically.

$$f(x) = 3x - 5$$

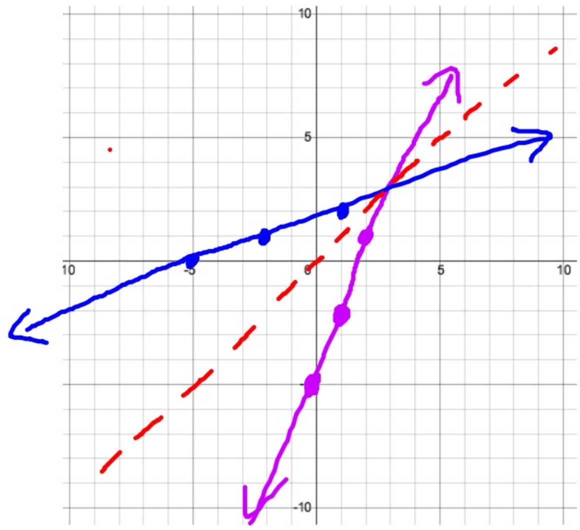
$$x = 3y - 5$$

$$x + 5 = 3y$$

$$\frac{x+5}{3} = y$$

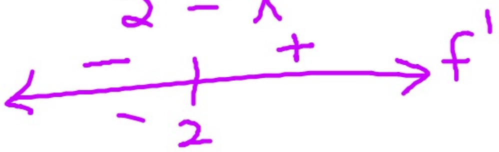
$$f^{-1}(x) = \frac{x+5}{3}$$

x	f(x)
0	-5
1	-2
2	1



Use the derivative to determine if the function is monotonic. Justify.

$$f(x) = x^2 - 4x + 1$$
$$f'(x) = 2x - 4$$
$$0 = 2x - 4$$
$$2 = x$$



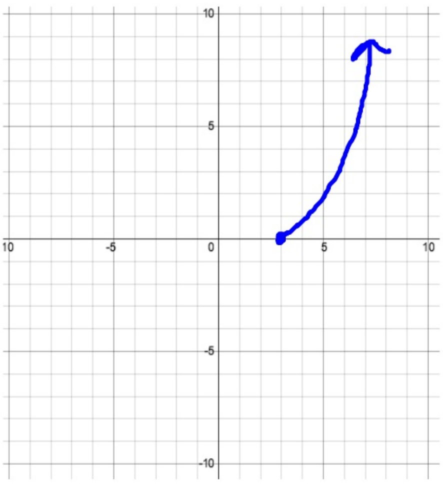
f(x) is not monotonic because
f'(x) < 0 on (-inf, 2) and
f'(x) > 0 on (2, inf)

$$g(x) = x^3 - 4$$
$$g'(x) = 3x^2$$
$$0 = 3x^2$$
$$0 = x$$

g(x) is monotonic because
g(x) is always increasing

$f(x) = (x-3)^2 \quad [3, \infty)$

Is this function monotonic? Explain.



$f'(x) = 2(x-3)$

$0 = 2(x-3)$

$3 = x$

$$\underset{3}{[}\xrightarrow{\quad + \quad} f'$$

f(x) is monotonic because f(x) is always increasing

f(x) is monotonic over the domain [3, in p.349 1-51eoo

1, 5, 9, 13, 17, 21, 25, 29, 33
37, 41, 45, 49