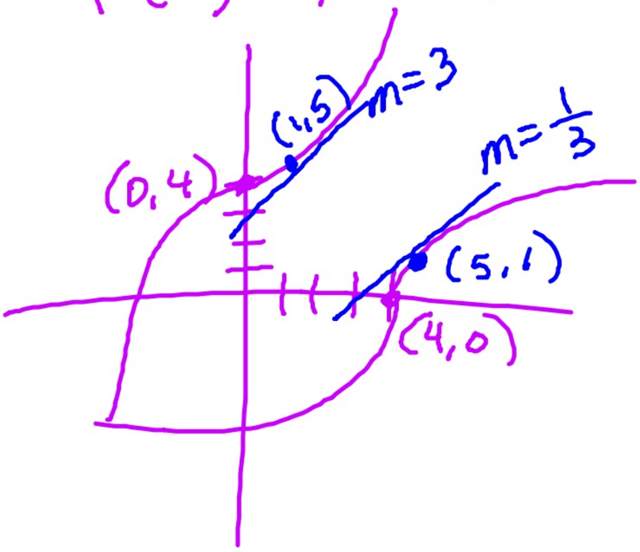
5.3 Inverse Functions

$f(x) = x^3 + 4$



$f^{-1}(x) = \sqrt[3]{x-4}$

$f'(x) = 3x^2 \quad f'(1) = 3$

$(f^{-1})'(x) = \frac{1}{3}(x-4)^{-2/3}$

$(f^{-1})'(5) = \frac{1}{3}(5-4)^{-2/3}$

$= \frac{1}{3}$

1. $f(x) = 2x^2 + 3x$ find $(f^{-1})'(5)$

$5 = 2x^2 + 3x$

$f: (1, 5)$

$x = 1$

$f^{-1}(5, 1)$

$f'(x) = 4x + 3$

$f'(1) = 7$

$(f^{-1})'(5) = \frac{1}{7}$

reciprocal

② $f(x) = x^3 + 2x - 1$ $\qquad (f^{-1})'(2)$

$2 = x^3 + 2x - 1$

$x = 1$

$0 = x^3 + 2x - 3$

Rat. zeros

$\pm 3, \pm 1$

$f: (1, 2)$

$f^{-1} (2, 1)$

$f'(x) = 3x^2 + 2$

$f'(1) = 5$

$(f^{-1})'(2) = \frac{1}{5}$

Let f be a function that is differentiable on the interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0$$

③ $f(x) = \frac{1}{3}x^3 + \frac{1}{3}$ and f and g are inverses. Find $g'(0)$.

$0 = \frac{1}{3}x^3 + \frac{1}{3}$

$-1 = x$

$f'(x) = x^2$

$f'(-1) = 1$

f: (-1, 0)

g: (0, -1)

$g'(0) = 1$

reciprocal

4

x	1	2	3	4
f	2	3	4	6
f'	6	5	2	3

f^{-1} $H(x)$: $(3,2)$

f : $(2,3)$

$H(x) = f^{-1}(x)$

Find $H'(3)$

$f'(2) = 5$

$(f^{-1})'(3) = \frac{1}{5}$

reciprocal