17.) $\int \frac{x^3 - 3x^2 + 5}{x-3}\,dx = \int \left(x^2 + \frac{5}{x-3}\right)dx$

$\frac{x^3}{3} + 5\ln|x-3| + C$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 0 & 5 \\ & & 3 & 0 & 0 \\ \hline & 1 & 0 & 0 & 5 \end{array}$$

53.) $\int_0^4 \frac{5}{3x+1}\,dx$

$u = 3x+1$

$du = 3dx$

$\frac{du}{3} = dx$

$\int \frac{5}{u} \cdot \frac{du}{3}$

$\frac{5}{3}\int \frac{1}{u}\,du = \frac{5}{3}\ln|u| = \frac{5}{3}\ln|3x+1|\Big|_0^4$

$\frac{5}{3}(\ln 13)$

13.) $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$

$u = x^3+3x^2+9x$

$du = 3x^2+6x+9\,dx$

$du = 3(x^2+2x+3)\,dx$

$\frac{du}{3(x^2+2x+3)} = dx$

$\frac{1}{3}\int \frac{1}{u}\,du$

$\frac{1}{3}\ln|x^3+3x^2+9x| + C$

11.) $\int \frac{x^2-4}{x}\,dx = \int \frac{x^2}{x} - \frac{4}{x}\,dx$

$\int \left(x - \frac{4}{x}\right)dx$

$\frac{1}{2}x^2 - 4\ln|x| + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

54.) $\int_{-1}^{1} \frac{1}{2x+3}\,dx$

$u = 2x+3$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| = \frac{1}{2}\ln|2x+3|\Big|_{-1}^{1}$

$\frac{1}{2}(\ln 5 - \ln 1)$

$\ln 1 - \ln 2$

$\ln\frac{1}{2}$

$0 - \ln 2 = \ln 2^{-1}$

$\frac{1}{2}\ln 5 = \ln\sqrt{5}$

Anti-Derivatives of tanx, cotx, secx, cscx

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$\frac{du}{-\sin x} = dx$

$$-\int \frac{1}{u}\,du$$

$$-\ln|u| + C = -\ln|\cos x| + C$$

Check: $-\left(\frac{-\sin x}{\cos x}\right) = \tan x$

$$\int \cot x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{u} du$$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

INTEGRALS OF THE SIX BASIC TRIGONOMETRIC FUNCTIONS

$$\int \sin u\, du = -\cos u + C \qquad \int \cos u\, du = \sin u + C$$

$$\int \tan u\, du = -\ln|\cos u| + C \qquad \int \cot u\, du = \ln|\sin u| + C$$

$$\int \sec u\, du = \ln|\sec u + \tan u| + C \qquad \int \csc u\, du = -\ln|\csc u + \cot u| + C$$

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1.) $\int \tan 7x\,dx$

$u = 7x$

$du = 7dx$

$\frac{du}{7} = dx$

$\int \tan u \cdot \frac{du}{7}$

$\frac{1}{7}\int \tan u\,du$

$-\frac{1}{7}\ln|\cos u| + C = -\frac{1}{7}\ln|\cos 7x| + C$

2.) $\int \sec\frac{x}{2}\,dx$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2\,du = dx$

$2\int \sec u\,du$

$2\ln|\sec u + \tan u| + C$

$2\ln\left|\sec\frac{x}{2} + \tan\frac{x}{2}\right| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

3.) $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}\,dx$

$u = 1+\sin x$

$du = \cos x\,dx$

$\int \frac{1}{u}\,du = \ln|1+\sin x|\Big|_0^{\pi/2}$

$\ln 2 - \ln 1$

$\ln 2$

Solve the differential equation.

$$\frac{dy}{dx} = \frac{1}{x-2} \qquad (4, 1)$$

$$y = \ln|x-2| + C$$

$$1 = \ln 2 + C$$

$$1 - \ln 2 = C$$

$$y = \ln|x-2| + 1 - \ln 2$$

$$y = \ln\left|\frac{x-2}{2}\right| + 1$$