$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$n \neq -1$$

$$\int x^{-1} dx = \int \frac{1}{x} dx$$

5.2 The Natural Logarithmic Function: Integration

THEOREM 5.5 LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

#1 $\int \frac{10}{x}\,dx = 10\int \frac{1}{x}\,dx$

$10\ln|x| + C$

When the degree on the denominator is larger, try u-substitution

#2 $\int \frac{1}{4-3x}\,dx$

$u = 4-3x$

$du = -3dx$

$\frac{du}{-3} = dx$

$\int \frac{1}{u}\frac{du}{-3}$

$-\frac{1}{3}\int \frac{1}{u}\,du = -\frac{1}{3}\ln|u| + C$

$= -\frac{1}{3}\ln|4-3x| + C$

$-\frac{1}{3}\cdot\frac{-3}{4-3x} = \frac{1}{4-3x}$

#3 $\int_1^2 \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \frac{x}{u}\cdot\frac{du}{2x} = \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|x^2+1|\Big|_1^2$$

$$= \frac{1}{2}(\ln 5 - \ln 2) = \frac{1}{2}\ln\frac{5}{2} = \ln\sqrt{\frac{5}{2}}$$

#4 $\int \frac{x^2-2x}{x^3-3x^2}\,dx$

$u = x^3-3x^2$

$du = 3x^2-6x\,dx$

$du = 3(x^2-2x)\,dx$

$\frac{du}{3(x^2-2x)} = dx$

$\int \frac{x^2-2x}{u} \cdot \frac{du}{3(x^2-2x)}$

$\frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|x^3-3x^2| + C$

When the degree of the numerator is greater than or equal to the degree of the denominator, try division.

Long Division (binomial in den.)

Synthetic (binomial in den.)

Breaking up the fraction (monomial in den.)

$$\frac{x^2+x-1}{x} = \frac{x^2}{x} + \frac{x}{x} - \frac{1}{x} = x + 1 - \frac{1}{x}$$

#5 $\int \frac{x+1}{x}\,dx = \int \left(\frac{x}{x} + \frac{1}{x}\right)dx$

$= \int \left(1 + \frac{1}{x}\right)dx = \int 1\,dx + \int \frac{1}{x}\,dx$

$= x + \ln|x| + C$

#6 $\int \frac{2x^2 + 7x - 3}{x - 2}\,dx = \int \left(2x + 11 + \frac{19}{x-2}\right)dx$

$$\begin{array}{r|rrr} 2 & 2 & 7 & -3 \\ & \downarrow & 4 & 22 \\ \hline & 2 & 11 & 19 \end{array}$$

$= \int 2x\,dx + \int 11\,dx + \int \frac{19}{x-2}\,dx$

$= x^2 + 11x + 19\int \frac{1}{x-2}\,dx$

$= x^2 + 11x + 19\ln|x-2| + C$

#7 $\int \frac{3x^2 + x - 1}{x+4}\,dx = \int 3x\,dx + \int -11\,dx + \int \frac{43}{x+4}\,dx$

$= \frac{3x^2}{2} - 11x + 43\ln|x+4| + C$

$-4 \mid 3 \quad 1 \quad -(\quad$

$\quad\quad -12 \quad 44$

$\quad 3 \quad -11 \quad 43$

5.2

1-170dd

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