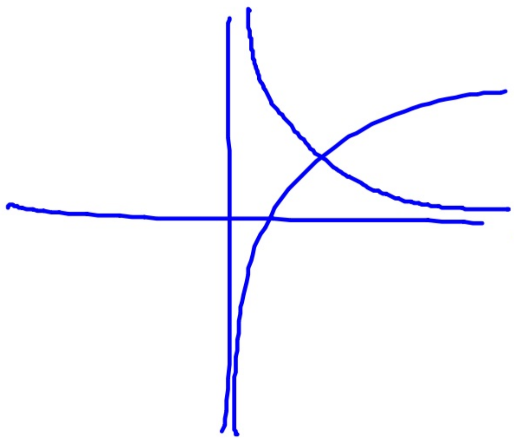
5.1 The Natural Logarithmic Function: Differentiation

- Develop and use properties of the natural logarithmic function.
- Understand the definition of the number e.
- Find derivatives of functions involving the natural logarithmic function.

Derivative of y = lnx

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$$



Derivative of y = ln u

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}, \quad u > 0$$

$$y = \ln(2x+3)$$

$$y' = \frac{1}{2x+3} \cdot 2 = \frac{2}{2x+3}$$

THEOREM 5.4 DERIVATIVE INVOLVING ABSOLUTE VALUE

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{u'}{u}.$$

Differentiate

#1 $f(x) = \ln|\cos x|.$

$$f'(x) = -\frac{\sin x}{\cos x}$$

$$f'(x) = -\tan x$$

② $y = \ln x^4$

$y = 4\ln x$

$y' = 4 \cdot \frac{1}{x}$

$y' = \frac{4}{x}$

③ $y = (\ln x)^4$

$y' = 4(\ln x)^3 \cdot \frac{1}{x}$

$y' = \frac{4(\ln x)^3}{x}$

④ $y = \ln\left(\frac{x^2+1}{2x-1}\right)$

$$y = \ln(x^2+1) - \ln(2x-1)$$

$$y' = \frac{2x}{x^2+1} - \frac{2}{2x-1}$$

5 $y = \ln\left(x^2\sqrt{3x+1}\right)$

$y = 2\ln x + \frac{1}{2}\ln(3x+1)$

$y' = 2 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{3}{3x+1}$

$y' = \frac{2}{x} + \frac{3}{6x+2}$

6 $y = x^4 \ln x$

$$y' = x^4 \cdot \frac{1}{x} + \ln x \cdot 4x^3$$

$$y' = x^3 + 4x^3 \ln x$$

⑦ $y = \ln\left|\frac{\cos x}{1+\cos x}\right|$

$y = \ln|\cos x| - \ln|1+\cos x|$

$y' = \frac{-\sin x}{\cos x} - \frac{-\sin x}{1+\cos x}$

$= -\tan x + \frac{\sin x}{1+\cos x}$

p. 332
47–65 odd
(skip 61)