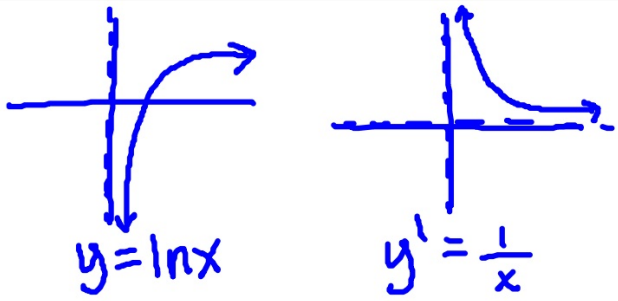
5.1 The Natural Logarithmic Function: Differentiation

- Develop and use properties of the natural logarithmic function.
- Understand the definition of the number e.
- Find derivatives of functions involving the natural logarithmic function.

Pg. 331: 77a, 81a, 84, 85, 91, 93, 101

Derivative of y = lnx

$$\frac{d}{dx}[\ln x] = \frac{1}{x} \quad x > 0$$



Derivative of y = ln u

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u} \quad u > 0$$

$y = \ln(3x+4)$

$y' = \frac{3}{3x+4}$

$$\ln 1 = 0$$

$$\ln e = 1$$

$$e^{\ln x} = x$$

$$\ln e^{x} = x$$

$$2^{\log_2 8}$$

Find the derivative.

#1 $h(t) = \dfrac{\ln t}{t}$

$$h'(t) = \frac{t \cdot \frac{1}{t} - \ln t \cdot 1}{t^2}$$

$$= \frac{1 - \ln t}{t^2}$$

#2 $y = \ln \sqrt[3]{\dfrac{x-1}{x+1}}$

$$y = \frac{1}{3} \ln\left(\frac{x-1}{x+1}\right)$$

$$y = \frac{1}{3}\left(\ln(x-1) - \ln(x+1)\right)$$

$$y' = \frac{1}{3}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

#3: Find the equation of the tangent line at the given point.

$$f(x) = 3x^2 - \ln x, \quad (1, 3)$$

$f'(x) = 6x - \frac{1}{x}$

$f'(1) = 5$

$y - 3 = 5(x - 1)$

#4: Find the relative extrema. Justify.

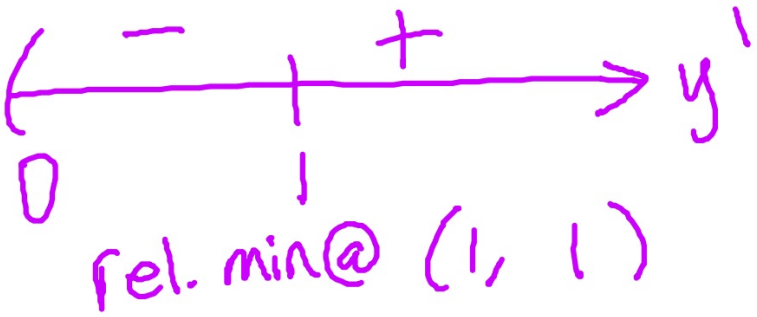
$$y = x - \ln x$$

D : (0, ∞)

$$y' = 1 - \frac{1}{x}$$

$$y' = \frac{x-1}{x}$$

crit : x = 1



rel. min @ (1, 1)

Relative minimum at (1, 1) because y' goes from negative to positive at this point

#5: Find dy/dx using implicit differentiation.

$$3x^2 + \ln x^2 y - y = 0$$

$$\left(3x^2 + 2\ln x + \ln y - y = 0\right)\frac{d}{dx}$$

$$6x\frac{dx}{dx} + 2\cdot\frac{1}{x}\frac{dx}{dx} + \frac{1}{y}\cdot\frac{dy}{dx} - 1\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(\frac{1}{y} - 1\right) = -6x - \frac{2}{x}$$

$$\frac{dy}{dx} = \frac{\left(-6x - \frac{2}{x}\right)xy}{\left(\frac{1}{y} - 1\right)xy} = \frac{-6x^2y - 2y}{x - xy}$$

#6 Find the derivative using logarithmic differentiation.

$$\ln y = \ln \frac{\sqrt{x+1}}{x}$$

$$\left(\ln y = \frac{1}{2}\ln(x+1) - \ln x\right)\frac{d}{dx}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{x+1} - \frac{1}{x}$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\cdot\frac{1}{x+1} - \frac{1}{x}\right)\cdot y$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\cdot\frac{1}{x+1} - \frac{1}{x}\right)\frac{\sqrt{x+1}}{x}$$

$$\frac{dy}{dx} = \frac{x-2(x+1)}{2x(x+1)}\cdot\frac{\sqrt{x+1}}{x}$$

$$= \frac{-x-2}{2x^2(x+1)}\cdot(x+1)^{1/2}$$

$$= \frac{-x-2}{2x^2\sqrt{x+1}}$$