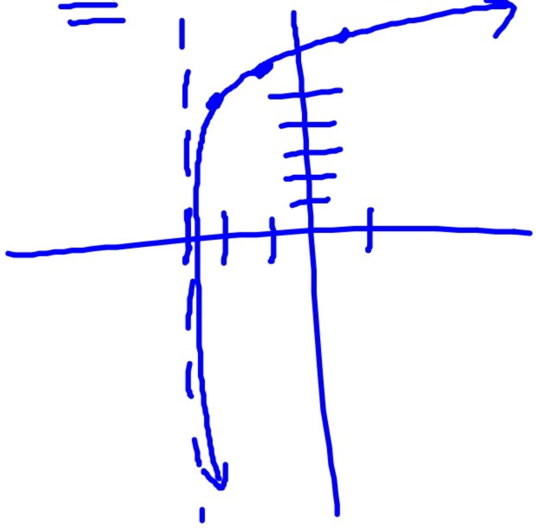
14.) $\log_3 \frac{8}{3} = \log_3 8 - \log_3 3$

$1.9 - 1$

10.) $y = \log_2 (x+3) + \underline{\underline{5}}$ D: $(-3, \infty)$

x	y
-2	5
-1	6
1	7



18.) $\dfrac{\cancel{8}\cdot 18^{x-7}}{\cancel{8}} = \dfrac{86}{8}$

$$\log\left(18^{x-7}\right) = \log\left(\frac{43}{4}\right)$$

$$(x-7)\log 18 = \log \frac{43}{4}$$

$$x \cancel{-7} = \frac{\log \frac{43}{4}}{\log 18} + 7$$

$$x = \log_{18} \frac{43}{4} + 7$$

20.) $\ln\sqrt{x+2} = 1$

$\frac{1}{2}\ln(x+2) = 1$

$e^{\ln(x+2)} = e^2$

$x+2 = e^2$

$x = e^2 - 2$

$e^{\ln x} = x$

$\ln e = 1$

$\ln 1 = 0$

19) $-6+3e^x=8$

$3e^x=14$

$\ln(e^x)=\ln\left(\frac{14}{3}\right)$

$x\cdot\cancel{\ln e}=\ln\frac{14}{3}$

$x=\ln\frac{14}{3}$

5.1 The Natural Logarithmic Function: Differentiation

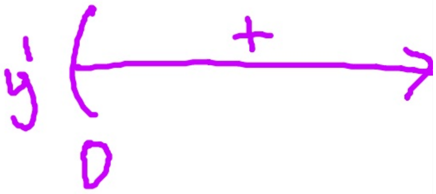
- Develop and use properties of the natural logarithmic function.
- Understand the definition of the number *e*.
- Find derivatives of functions involving the natural logarithmic function.

Derivative of y = lnx

$$\frac{d}{dx}[\ln x] = \frac{1}{x} \qquad x > 0$$

Derivative of y = ln u

$$\frac{d}{dx}[\ln(u)] = \frac{1}{u}\,\frac{du}{dx} = \frac{u'}{u} \qquad u > 0$$



$y = \sin 4x$

$y' = \cos 4x \cdot 4$

$y = \sin u$

$y' = \cos u \cdot u'$

$\frac{1}{u} \cdot u'$

① $y = \ln(3x+4)$

$$y' = \frac{3}{3x+4}$$

$$\frac{u'}{u}$$

$$\frac{1}{u} \cdot u'$$

---

② $y = \ln\left(x(2x+1)^4\right)$

$$y = \ln x + 4\ln(2x+1)$$

$$y' = \frac{1}{x} + 4 \cdot \frac{2}{2x+1} = \frac{1}{x} + \frac{8}{2x+1}$$

Find the derivative.

#3 $h(t) = \frac{\ln t}{t}$

$$h'(t) = \frac{t \cdot \frac{1}{t} - \ln t \cdot 1}{t^2}$$

$$h'(t) = \frac{1 - \ln t}{t^2}$$

#4 $f(x) = \ln \sqrt[3]{\frac{2x+1}{x^2+1}}$

$$f(x) = \frac{1}{3}\left[\ln(2x+1) - \ln(x^2+1)\right]$$

$$f'(x) = \frac{1}{3}\left[\frac{2}{2x+1} - \frac{2x}{x^2+1}\right]$$

THEOREM 5.4 DERIVATIVE INVOLVING ABSOLUTE VALUE

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{u'}{u}$$

#5 $f(x) = \ln|\cos x|$.

$$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$$

45.) $y = \ln x^4$

#6 Write the equation of the tangent line at (1,0) for $f(x) = \ln\sqrt{x}$

$$f(x) = \frac{1}{2}\ln x$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$$

$$f'(1) = \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x - 1)$$

$$(\ln x)^2 \neq \ln x^2$$

$$2\ln x$$

$$y = (\ln x)^2$$

$$y' = 2(\ln x)^1 \cdot \frac{1}{x}$$

$$y' = \frac{2\ln x}{x}$$

$$y = \ln x^2$$

$$y = 2\ln x$$

$$y' = \frac{2}{x}$$

57.) $f(x) = \ln x - \ln(x^2+1)$

$$f'(x) = \frac{1}{x} - \frac{2x}{x^2+1}$$

45.) $y = \ln x^4$

(1,0) $y = 4 \cdot \ln x$

$y' = 4 \cdot \frac{1}{x}$ $\quad y - 0 = 4(x-1)$

$y'(1) = 4$

61.) $y = \ln(\ln x^2) = \ln(2 \cdot \ln x)$

$$y' = \frac{\frac{2}{x}}{2\ln x}$$

$$y' = \frac{2}{2x\ln x}$$

$$y' = \frac{1}{x\ln x}$$

$$\frac{u'}{u}$$

$u = 2\ln x$

$du = \frac{2}{x}$

59)

$$g(t) = \frac{\ln t}{t^2}$$

$$g'(t) = \frac{t^2 \cdot \frac{1}{t} - \ln t \cdot 2t}{t^4}$$

$$= \frac{t - 2t \ln t}{t^4} = \frac{t(1 - 2\ln t)}{t^4}$$

$$= \frac{1 - 2\ln t}{t^3}$$

#7 Write the equation of the tangent line at (1, 0)

$f(x) = \frac{1}{2}x \ln x^2$ $\quad$ $\frac{1}{\cancel{2}}x \cdot \cancel{2} \ln x$

$f(x) = x \ln x$

$f'(x) = x \cdot \frac{1}{x} + \ln x \cdot 1$

$f'(x) = 1 + \ln x$

$f'(1) = 1$

$y - 0 = 1(x - 1)$

$y = x - 1$

#9: Find the relative extrema. Justify.

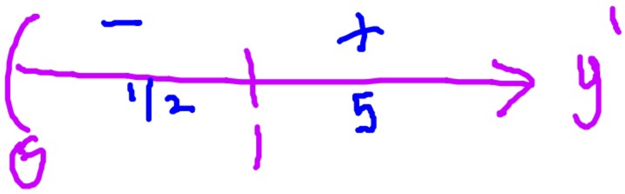
$$y = x - \ln x$$

D: $(0, \infty)$

$$y' = 1 - \frac{1}{x}$$

$$y' = \frac{x-1}{x}$$

$x = 1$



relative min @ $(1, 1)$ because y' changes from $-/+$ at this point.

#91 rel. extrema

#8: Find dy/dx using implicit differentiation.

$$4xy + \ln x^2y = 7$$

$$\frac{d}{dx}\left(4xy + 2\ln x + \ln y = 7\right)$$

$$4x\frac{dy}{dx} + y\cdot 4\frac{dx}{dx} + \frac{2}{x}\frac{dx}{dx} + \frac{1}{y}\frac{dy}{dx} = 0$$

$$4x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = -4y - \frac{2}{x}$$

$$\frac{dy}{dx}\left(4x + \frac{1}{y}\right) = -4y - \frac{2}{x}$$

$$\frac{dy}{dx} = \frac{\left(-4y - \frac{2}{x}\right)xy}{\left(4x + \frac{1}{y}\right)xy} = \frac{-4xy^2 - 2y}{4x^2y + x}$$