# 4.5 Integration with Substitution Part 2

Change of plans...

1) Today is the last lesson of Chapter 4
2) The quiz will be Friday; the test will be Tuesday

HW: Pg. 307 47, 49, 55, 61, 63, 79, 82, 85, 95

#1 $\int x \sin x^2 \, dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \cancel{x} \sin u \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \sin u \, du$

$-\frac{1}{2}\cos u + C$

$-\frac{1}{2}\cos x^2 + C$

#2 $\int_0^{\pi/6} \cos(3x)\, dx$

$u = 3x$

$du = 3dx$

$\frac{du}{3} = dx$

$\int \cos u \frac{du}{3}$

$\frac{1}{3}\sin u$

$\frac{1}{3}\sin 3x \Big|_0^{\pi/6}$

$\frac{1}{3}(1-0) = \frac{1}{3}$

#3 $\int \tan x \sec^2 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$\int u\,du = \frac{u^2}{2} + C$

$= \frac{\tan^2 x}{2} + C$

#4 $\int \sin^4 x \cos x\,dx$

$\int (\sin x)^4 \cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\int u^4\,du$

$\frac{u^5}{5} + C$

$\frac{\sin^5 x}{5} + C$

#5

$$\int_{\pi/4}^{\pi/3} \sec^2(3x)\,dx$$

$$\int \sec^2\left(\frac{x}{2}\right)dx$$

$u = 3x$

$du = 3dx$

$\frac{du}{3} = dx$

$$\int \sec^2 u \frac{du}{3} = \frac{1}{3}\int \sec^2 u\,du$$

$$\frac{1}{3}\tan u = \frac{1}{3}\tan 3x \Big|_{\pi/4}^{\pi/3}$$

$$\frac{1}{3}\left(\tan\pi - \tan\frac{3\pi}{4}\right) = \frac{1}{3}(0 - -1)$$

$1/3$

82) $\int_0^2 x\sqrt[3]{4+x^2}\,dx$

$u = 4 + x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int x\sqrt[3]{u}\,\frac{du}{2x}$

$\frac{1}{2}\int u^{1/3}\,du = \frac{1}{2}\cdot\frac{u^{4/3}}{4/3} = \frac{3}{8}\left(4+x^2\right)^{4/3}\Big|_0^2$

$= \frac{3}{8}\left(8^{4/3} - 4^{4/3}\right) = \frac{3}{8}\left(16 - \sqrt[3]{256}\right)$

$\frac{3}{2}\left(4 - \sqrt[3]{4}\right) = \frac{3}{8}\left(16 - 4\sqrt[3]{4}\right)$

47.) $\int \pi \sin(\pi x)\,dx$

$u = \pi x$

$du = \pi\,dx$

$\int \sin \pi x \cdot \pi\,dx$

$\int \sin u\,du = -\cos u + C$

$= -\cos \pi x + C$

61.) $f'(x) = -\sin\frac{x}{2}$ $\quad (0, 6)$

$f(x) = \int -\sin\frac{x}{2}\,dx$

$u = \frac{x}{2} = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2du = dx$

$= -\int \sin u \cdot 2du$

$= -2\int \sin u\,du$

$f(x) = 2\cos u + C$

$f(x) = 2\cos\left(\frac{x}{2}\right) + C$

$6 = 2\cos 0 + C$

$4 = C$

$\boxed{f(x) = 2\cos\frac{x}{2} + 4}$

① $\int \frac{x}{(1-x^2)^4}dx$

$u = 1 - x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$-\frac{1}{2}\int u^{-4}du$

$-\frac{1}{2}\cdot\frac{u^{-3}}{-3} = \frac{1}{6}(1-x^2)^{-3} + C$

② $\int_0^1 \sqrt{3x+1}\,dx$

$u = 3x+1$

$du = 3dx$

$\frac{du}{3} = dx$

$\frac{1}{3}\int u^{1/2}du$

$\frac{1}{3}\cdot\frac{u^{3/2}}{3/2}$

$\frac{2}{9}(3x+1)^{3/2}\Big|_0^1$

$\frac{2}{9}(4^{3/2}-1) = \boxed{\frac{14}{9}}$

③ $\int \cos 7x \, dx$

④ $\int \sin^4 x \cos x \, dx$

⑤ $\int \cot x \csc^2 x \, dx$

⑥ $\int_0^{\pi/2} (1 - \sin 2x) \, dx$

$= \int_0^{\pi/2} 1 \, dx - \int_0^{\pi/2} \sin 2x \, dx$

③ $\int \cos 7x \, dx$

$\frac{1}{7}\sin 7x + C$

④ $\int \sin^4 x \cos x \, dx$

$\int u^4 du = \frac{1}{5}u^5 + C$

$\frac{1}{5}\sin^5 x + C$

⑤ $\int \cot x \csc^2 x \, dx$

$u = \cot x$

$du = -\csc^2 x dx$

$-\int u \, du$

$-\frac{1}{2}u^2 + C = -\frac{1}{2}(\cot x)^2 + C$

⑥ $\int_0^{\pi/2} (1 - \sin(2x)) dx$

$x + \frac{1}{2}\cos 2x \Big|_0^{\pi/2}$

$\left(\frac{\pi}{2} + -\frac{1}{2}\right) - \left(0 + \frac{1}{2}\right) = \frac{\pi}{2} - 1$