80.) $\frac{3}{16}$

82.) $32\frac{2}{3}$

88.) $= -\frac{1}{3}$

90.) $\frac{1}{4}$

57.) $\int \frac{\cos 2x}{(1+\sin 2x)^2}\,dx$

$u = 1+\sin 2x$

$\frac{du}{2} = \frac{\cancel{2}\cos 2x\,dx}{\cancel{2}}$

$\frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\cdot\frac{u^{-1}}{-1} + C$

$= -\frac{1}{2u} + C = \frac{-1}{2(1+\sin 2x)} + C$

55.) $\int \sec^2(4x+9)\,dx$

$\int \sec^2 x\,dx = \tan x + C$

$u = 4x+9$

$\frac{du}{4} = \frac{\cancel{4}dx}{\cancel{4}}$

$\frac{du}{4} = dx$

$\frac{1}{4}\int \sec^2 u\,du$

$\frac{1}{4}\tan u + C$

$\frac{1}{4}\tan(4x+9) + C$

83.) $\int_0^2 \frac{x+3}{(x^2+6x+1)^3}dx = \frac{1}{2}\int u^{-3}du$

$u = x^2+6x+1$

$du = \frac{(2x+6)dx}{2}$

$\frac{du}{2}$

$\frac{du}{2} = (x+3)dx$

$\frac{1}{2}\cdot\frac{u^{-2}}{-2} = \frac{1}{-4(x^2+6x+1)^2}\Big]_0^2$

$-\frac{1}{4}\left[\frac{1}{17^2} - \frac{1}{1}\right]$

$-\frac{1}{4}\left[\frac{1}{289} - 1\right]$

$-\frac{1}{4}\left[\frac{-288}{289}\right]$

$-\frac{1}{4}\left(\frac{1}{17^2}\right) + \frac{+1}{4}(1)$

$-\frac{1}{4}\left(\frac{1}{17^2} - \frac{1}{\cancel{4}}\right)$

$\frac{72}{289}$

4.6 More U Substitution

Solve the differential equation

#1 $\int \frac{dy}{\cancel{dx}} = \int \frac{\cancel{10}x^2}{\sqrt{1+x^3}}\, dx$

$u = 1 + x^3$

$\frac{du}{3} = \frac{\cancel{3}x^2 dx}{\cancel{3}}$

$\frac{du}{3} = x^2 dx$

$y = 10\int \frac{x^2}{\sqrt{1+x^3}}\, dx$

$y = \frac{10}{3}\int u^{-1/2}\, du = \frac{10}{3} \cdot \frac{u^{1/2}}{1/2} + C$

$\boxed{y = \frac{20}{3}\sqrt{1+x^3} + C}$

#2 $\int \frac{dy}{\cancel{dx}} = -2\int \sec(2x)\tan(2x)\,dx$

$(0, -1)$

Find the particular solution to the differential equation.

$y = \frac{-2}{2}\int \sec(2x)\tan(2x)\,dx$

$u = 2x$

$\frac{du}{2} = \frac{\cancel{2}dx}{\cancel{2}}$

$y = -\int \sec u \tan u \, du$

$y = -\sec u + C$

$y = -\sec(2x) + C$

$-1 = -(1) + C$

$0 = C$

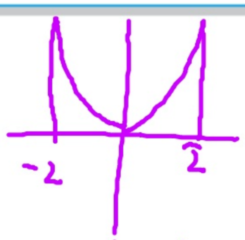
$\boxed{y = -\sec(2x)}$

THEOREM 4.16 INTEGRATION OF EVEN AND ODD FUNCTIONS

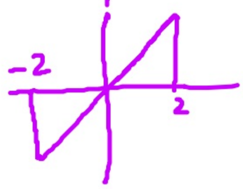
Let f be integrable on the closed interval $[-a, a]$.

✓1. If f is an *even* function, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$

✓2. If f is an *odd* function, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0.$



$$\int_{-2}^{2} x^2\,dx = 2\int_{0}^{2} x^2\,dx$$



$$\int_{-2}^{2} x\,dx = 0$$

$$\int_{-\pi/2}^{\pi/2} \sin x\,dx = 0$$

$y = f(x)$

$y = f(-x)$

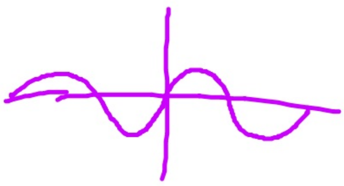
$y = -f(x)$

odd

$$y = \sin x$$

$$y(-x) = \sin(-x)$$

$$= -\sin x$$



$$-\cos x \Big]_{-\pi/2}^{\pi/2}$$

$$-(0-0)$$

$$0$$

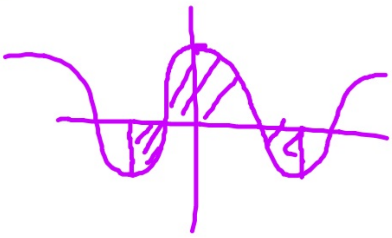
$$\int_{-\pi/2}^{\pi/2} \cos x \, dx = 2\int_{0}^{\pi/2} \cos x$$

$$= 2 \sin x \Big]_{0}^{\pi/2}$$

$$= 2$$

$y = \cos x$

$y(-x) = \cos(-x) = \cos x$



$y = f(x)$

$y(-x) = f(-x)$

$y = f(x)$

even

$$\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$$

$\frac{1}{2}$ $\frac{1}{2}$