4.5 Integration by Substitution

Sometimes, you will encounter a more complicated integrand. We can use u-substitution when there is a composite function. (think backwards chain rule)

$y = (1+x^2)^5$ $\quad$ $y' = 5(1+x^2)^4 \cdot 2x$

$\int 10x(1+x^2)^4\,dx$ $\quad$ $y' = 10x(1+x^2)^4$

#1

$$\int (x^2 - 9)^3 (2x)\, dx$$

$u = x^2 - 9$

$du = 2x\,dx$

$$\int u^3\, du = \frac{u^4}{4} + C$$

U is a temporary variable; plug back the value of u.

$$\boxed{\frac{(x^2-9)^4}{4} + C}$$ ✓

$\frac{\cancel{4}}{\cancel{4}}(x^2-9)^3 \cdot 2x$

#2 $$\int x^2(x^3+5)^4\,dx$$

$u = x^3 + 5$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$$\int (x^3+5)^4 x^2\,dx = \int u^4 \cancel{x^2} \frac{du}{3\cancel{x^2}}$$

$$\frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{u^5}{5} + C = \frac{(x^3+5)^5}{15} + C$$

#3

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx$$

$u = 1 + x^4$

$du = 4x^3\,dx$

$\frac{du}{4x^3} = dx$

$$\int \frac{\cancel{x^3}}{\sqrt{u}} \cdot \frac{du}{4\cancel{x^3}}$$

$$\frac{1}{4}\int u^{-1/2}\,du = \frac{1}{4} \cdot \frac{u^{1/2}}{1/2} + C$$

$$\frac{1}{2}\sqrt{1+x^4} + C$$

#4

$$\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$$

$u = 1 + 2x^2$

$du = 4x\,dx$

$\frac{du}{4x} = dx$

$$\int \frac{\cancel{x}}{\sqrt{u}} \cdot \frac{du}{4\cancel{x}}$$

$$\frac{1}{4}\int u^{-1/2}\,du = \frac{1}{4} \cdot \frac{u^{1/2}}{1/2} = \frac{1}{2}\sqrt{1+2x^2}\Big|_0^2$$

$$\frac{1}{2}\left(\sqrt{9} - \sqrt{1}\right) = 1$$

#5

$$\int \frac{t - 9t^2}{\sqrt{t}}\,dt \qquad \int t^{-1/2}(t - 9t^2)\,dt$$

$$\int (t^{1/2} - 9t^{3/2})\,dt = \frac{2}{3}t^{3/2} - \frac{9t^{5/2}}{5/2} + C$$

$$= \frac{2}{3}t^{3/2} - \frac{18}{5}t^{5/2} + C$$

#6 $\int \left(1+\frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$

$u = 1 + \frac{1}{t}$

$du = -t^{-2}\,dt$

$du = \frac{-1}{t^2}\,dt$

$-t^2\,du = dt$

$\int u^3 \frac{1}{\cancel{t^2}}\left(-\cancel{t^2}\,du\right)$

$-\int u^3\,du = -\frac{u^4}{4} + C = \frac{-1}{4}\left(1+\frac{1}{t}\right)^4 + C$

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