4.4 The Fundamental Theorem of Calculus

DEFINITION OF THE AVERAGE VALUE OF A FUNCTION ON AN INTERVAL

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

$$\frac{\int_a^b f(x)\,dx}{b-a}$$

FIND THE AVERAGE VALUE
ON THE CLOSED INTERVAL.

set up
first

#1 $4x^3 - 3x^2, \quad [-1, 2]$

$$\frac{1}{2--1}\int_{-1}^{2}(4x^3-3x^2)\,dx = \frac{1}{3}\left(x^4-x^3\right)\Big|_{-1}^{2}$$

$$\frac{1}{3}\left[\left(2^4-2^3\right)-\left((-1)^4-(-1)^3\right)\right]$$

$$\frac{1}{3}(8-2)=2$$

Find the average value on the interval.

#2 $f(x) = \cos x, \quad [0, \pi/2]$

$$\frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} \cos x \, dx = \frac{2}{\pi} \sin x \Big|_0^{\pi/2}$$

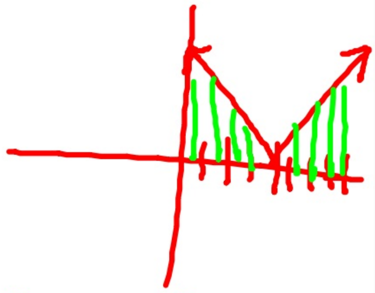
$$\frac{2}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$\frac{2}{\pi}$$

#4 $\int_0^8 |x-4|\,dx$

Use geometric shapes for absolute value functions

$\int_0^4 (-x+4)\,dx + \int_4^8 (x-4)\,dx$



$2\left(\frac{1}{2}bh\right)$

$2\left(\frac{1}{2}\cdot 4\cdot 4\right)$

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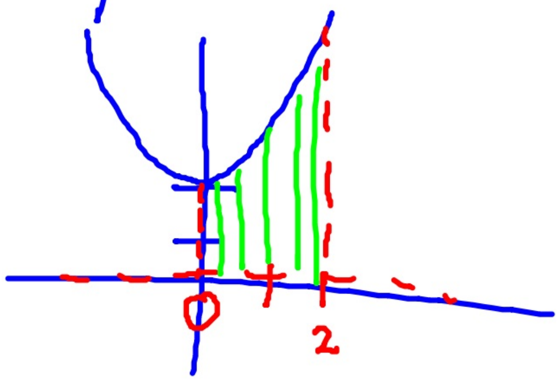
51, 53, 55

#3 Find the area under the curve on the given interval.

$y = 5x^2 + 2$; $x = 0, x = 2, y = 0$



$$\int_0^2 (5x^2 + 2)\,dx$$

$$= \frac{5x^3}{3} + 2x \Big|_0^2$$

$$\frac{40}{3} + 4 = \frac{52}{3}$$

13) $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx = \int_1^2 \left(3x^{-2} - 1\right) dx$

$\frac{3x^{-1}}{-1} - x = \frac{-3}{x} - x \Big|_1^2$

$\left(\frac{-3}{2} - 2\right) - \left(\frac{-3}{1} - 1\right)$

$\frac{-7}{2} - (-4) = -\frac{7}{2} + 4 = \frac{1}{2}$

31.) $\displaystyle\int_{-\pi/6}^{\pi/6} \sec^2 x \, dx$

$$\tan x \Big|_{-\pi/6}^{\pi/6}$$

$$\tan\frac{\pi}{6} - \left(\tan\left(-\frac{\pi}{6}\right)\right)$$

$$\frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3}$$