4.3 Riemann Sums and Definite Integrals

indefinite

$$\int (x+1)\,dx$$

answer is an expression

definite

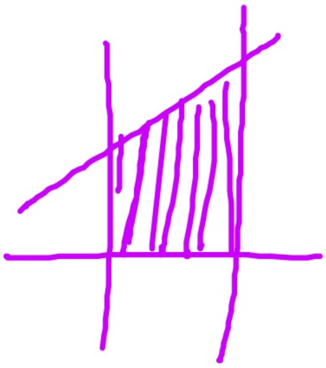
$$\int_2^5 (x+1)\,dx$$

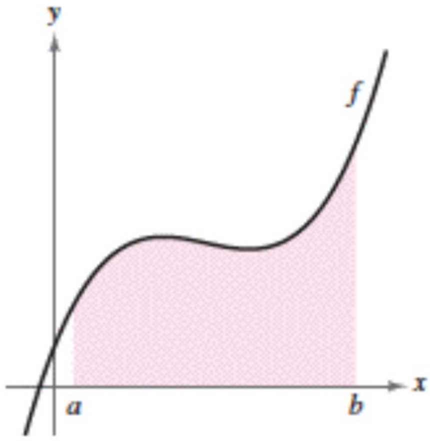
answer is a number

THEOREM 4.5 THE DEFINITE INTEGRAL AS THE AREA OF A REGION

If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b f(x)\,dx.$$

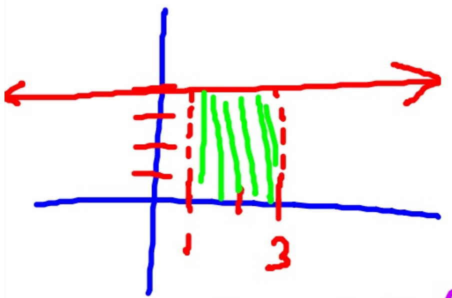




Sketch, then find the value of the definite integral.

#1 $\int_1^3 4\,dx = 8$

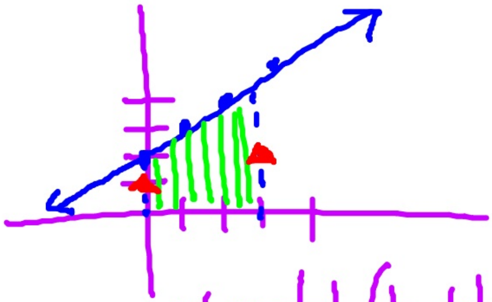
$f(x) = 4$



Area $= 8$

#2 $\int_0^3 (x+2)\,dx = \frac{21}{2}$

$f(x) = x + 2$



Trapezoid $= \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(3)(2+5)$

$= \frac{21}{2}$

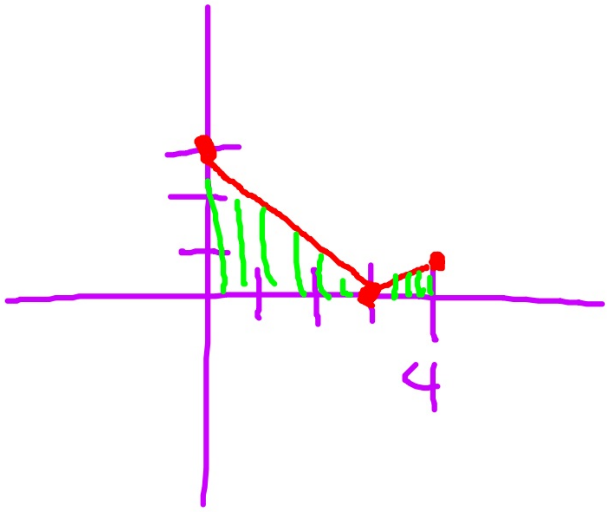
#3

$$\int_0^4 |x-3|\,dx$$

$\frac{1}{2}bh + \frac{1}{2}bh$

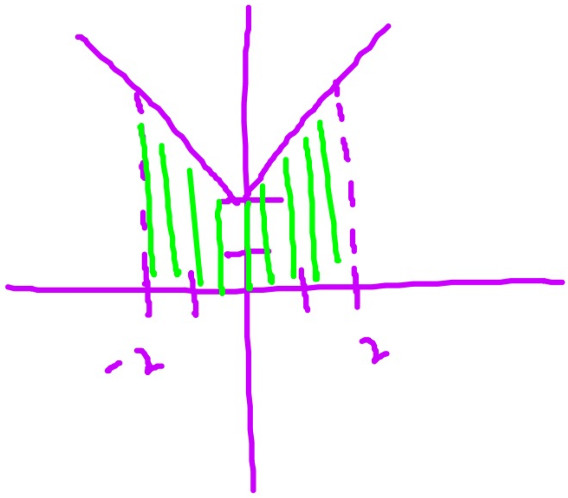
$\frac{1}{2}(3)(3) + \frac{1}{2}(1)(1)$

5



#4

$$\int_{-2}^{2} (|x| + 2)\,dx$$



$$\cancel{2}\left(\cancel{\tfrac{1}{2}} h \left(b_1 + b_2\right)\right)$$

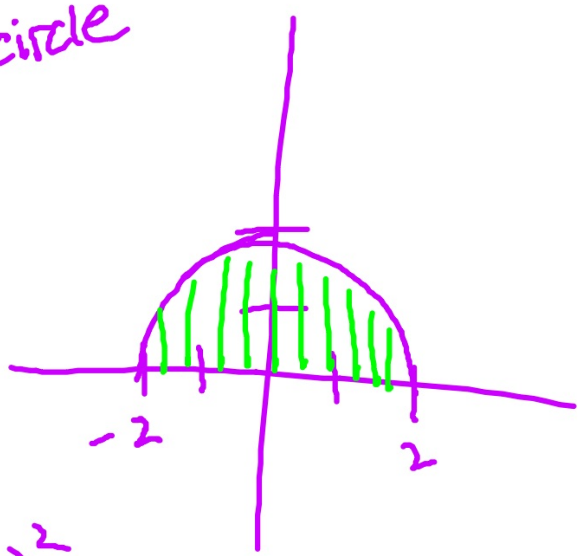
$$2(2+4)$$

$$12$$

#5 $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$ semi-circle

$x^2 + y^2 = 4$

$y = \pm\sqrt{4 - x^2}$



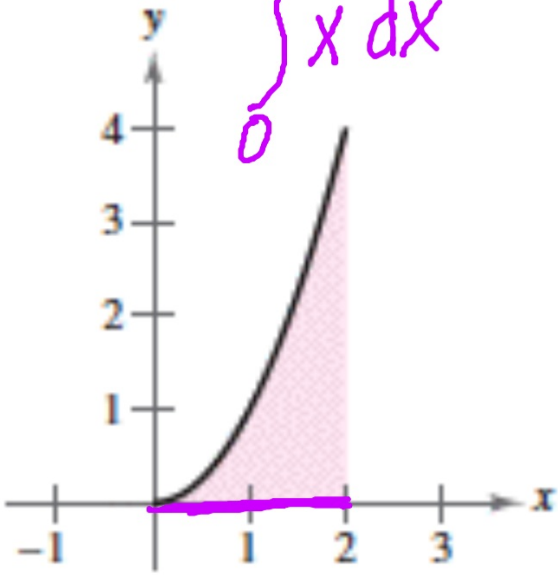
$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2$

$= 2\pi$

Set up a definite integral that would represent the area of the region. Do not evaluate.

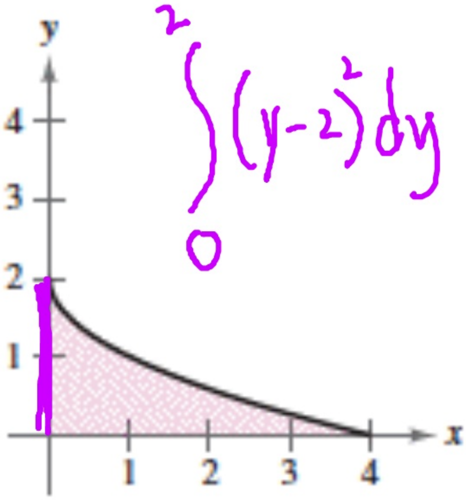
#6 $f(x) = x^2$

$\int_0^2 x^2\,dx$



#7 $f(y) = (y-2)^2$

$\int_0^2 (y-2)^2\,dy$



DEFINITIONS OF TWO SPECIAL DEFINITE INTEGRALS

1. If f is defined at $x = a$, then we define $\int_a^a f(x)\,dx = 0$.

2. If f is integrable on $[a, b]$, then we define $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$.

THEOREM 4.6 ADDITIVE INTERVAL PROPERTY

If f is integrable on the three closed intervals determined by a, b, and c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

THEOREM 4.7 PROPERTIES OF DEFINITE INTEGRALS

If f and g are integrable on $[a, b]$ and k is a constant, then the functions kf and $f \pm g$ are integrable on $[a, b]$, and

1. $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

2. $\displaystyle\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$

#1

Given $\int_{-1}^{1} f(x)\,dx = 0$ and $\int_{0}^{1} f(x)\,dx = 5$, evaluate

(a) $\displaystyle\int_{-1}^{0} f(x)\,dx.$ (b) $\displaystyle\int_{0}^{1} f(x)\,dx - \int_{-1}^{0} f(x)\,dx.$

(c) $\displaystyle\int_{-1}^{1} 3f(x)\,dx.$ (d) $\displaystyle\int_{0}^{1} 3f(x)\,dx.$

#2

Evaluate $\int_1^3 (-x^2 + 4x - 3)\, dx$ using each of the following values.

$$\int_1^3 x^2\, dx = \frac{26}{3}, \qquad \int_1^3 x\, dx = 4, \qquad \int_1^3 dx = 2$$