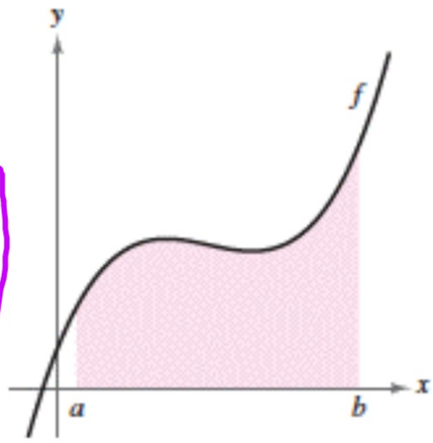
# 4.3 Riemann Sums and Definite Integrals

**THEOREM 4.5 THE DEFINITE INTEGRAL AS THE AREA OF A REGION**

If $f$ is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of $f$, the $x$-axis, and the vertical lines $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b f(x)\,dx.$$




$$\boxed{\int f'(x)\,dx = f(x) + C}$$

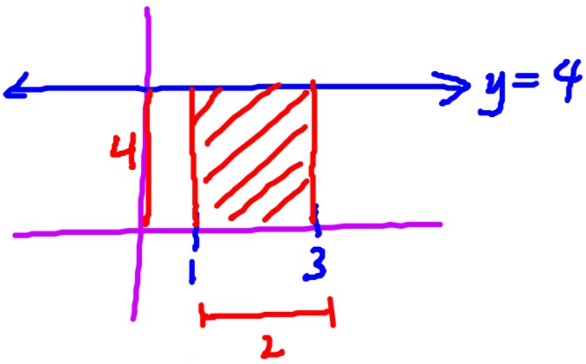
indefinite integral

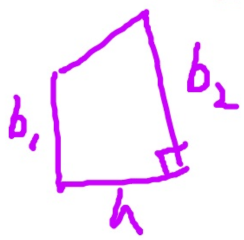
Sketch, then find the value of the definite integral.

#1

$$\int_1^3 4\,dx = 8$$

$f(x) = 4$

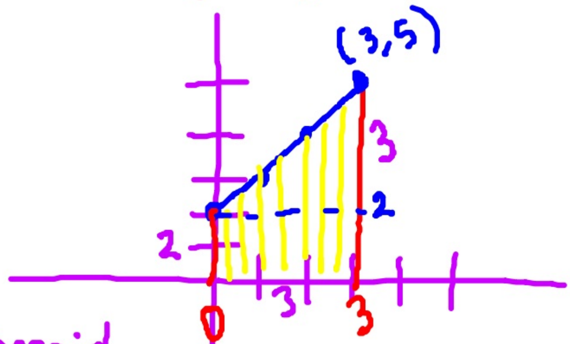



#2

$$\int_0^3 (x+2)\,dx$$

$f(x) = x + 2$




trapezoid

$\frac{1}{2}h(b_1+b_2)$

$\frac{1}{2}\cdot 3(2+5)$

$10.5$

rect : $3\cdot 2 = 6$

$\Delta$ : $\frac{1}{2}\cdot 3\cdot 3 = 4.5$

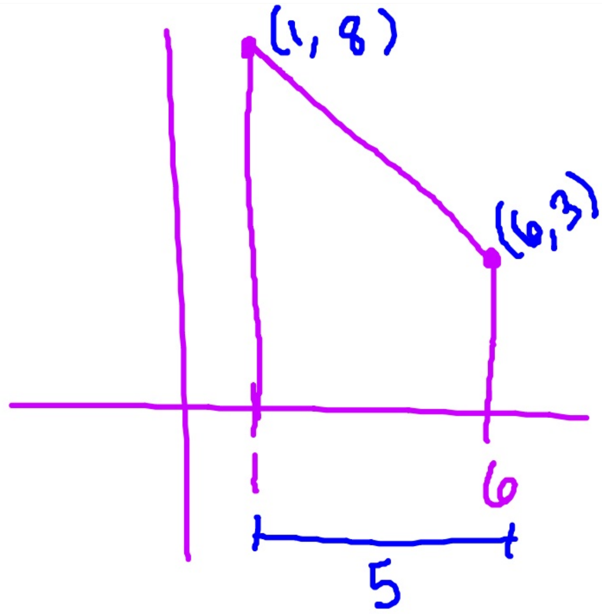
$10.5$

#3

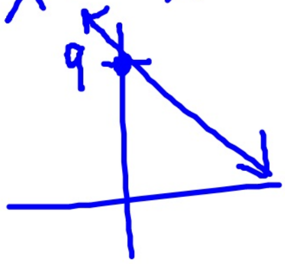
$$\int_1^6 (9-x)\,dx$$

$$\frac{1}{2}(5)(8+3)$$

$$\frac{55}{2}$$




$$y = 9 - x = -x + 9$$



#4 $\int_{-2}^{2} \sqrt{4-x^2}\,dx = 2\pi$

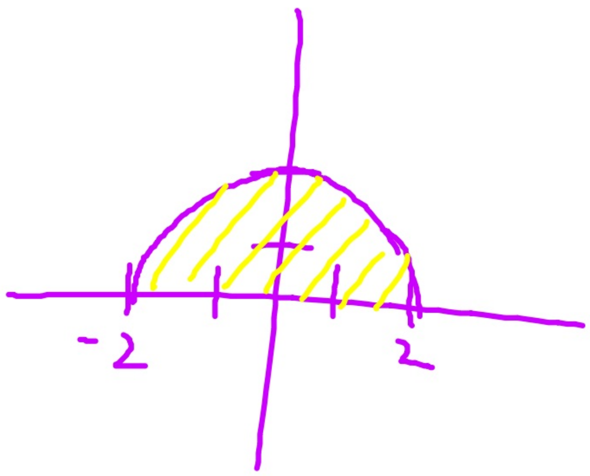
semicircle

$r = 2$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$

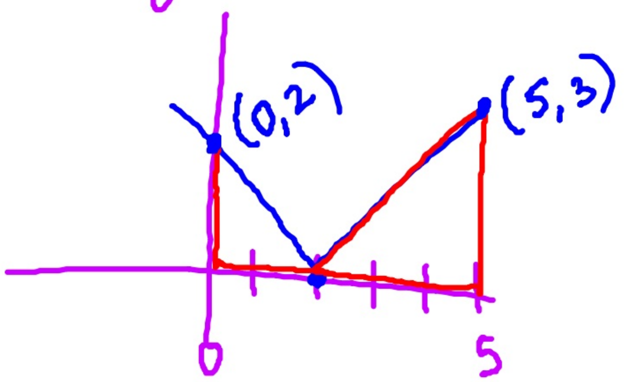



$\frac{1}{2}\pi r^2$

$\frac{1}{2}\pi 2^2$

$2\pi$

\# 5 $\int_0^5 |x-2|\,dx = 6.5$

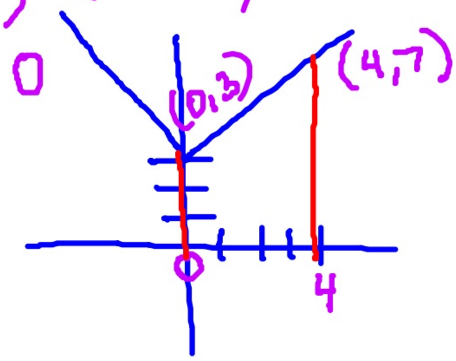



$\frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot 3\cdot 3$

$2 + 4.5$

$6.5$

\# 6 $\int_0^4 (|x|+3)\,dx = 20$

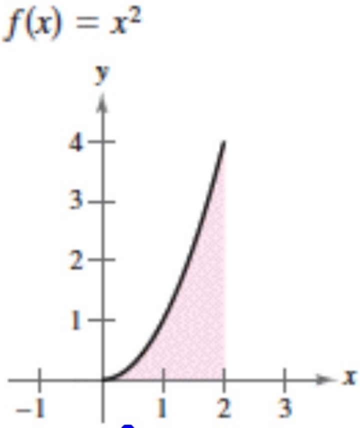



$\frac{1}{2}(4)(3+7)$

$20$

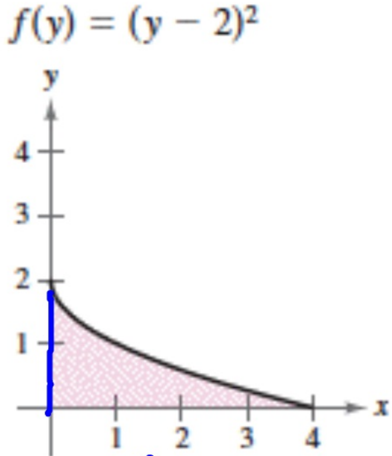
Set up a definite integral that would represent the area of the region. Do not evaluate.

#7




$$\int_0^2 x^2\,dx$$

#8




$$\int_0^2 (y-2)^2\,dy$$

## DEFINITIONS OF TWO SPECIAL DEFINITE INTEGRALS

1. If $f$ is defined at $x = a$, then we define $\int_a^a f(x)\,dx = 0$.

2. If $f$ is integrable on $[a, b]$, then we define $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$.

## THEOREM 4.6 ADDITIVE INTERVAL PROPERTY

If $f$ is integrable on the three closed intervals determined by $a$, $b$, and $c$, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

$$\int_2^3 f(x)dx = 10$$

$$\int_3^6 f(x)dx = 4$$

$$\int_2^6 f(x)dx = 14$$

**THEOREM 4.7 PROPERTIES OF DEFINITE INTEGRALS**

If $f$ and $g$ are integrable on $[a, b]$ and $k$ is a constant, then the functions $kf$ and $f \pm g$ are integrable on $[a, b]$, and

**1.** $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

**2.** $\displaystyle\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$

#1 Given $\int_{-1}^{1} f(x)\,dx = 0$ and $\int_{0}^{1} f(x)\,dx = 5$, evaluate

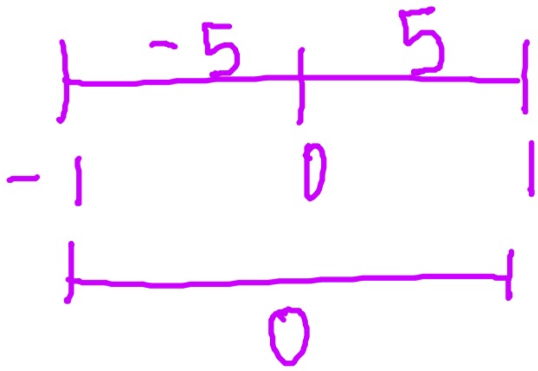
(a) $\int_{-1}^{0} f(x)\,dx. = -5$

(b) $\int_{0}^{1} f(x)\,dx - \int_{-1}^{0} f(x)\,dx.$

$5 - -5 = 10$

(c) $3\int_{-1}^{1} \cancel{3}f(x)\,dx. = 0$

(d) $\int_{0}^{1} 3f(x)\,dx. = 15$




$\int_{1}^{0} f(x)\,dx = -5$

#2 Evaluate $\int_1^3 (-x^2 + 4x - 3)\, dx$ using each of the following values.

$$\int_1^3 x^2\, dx = \frac{26}{3}, \qquad \int_1^3 x\, dx = 4, \qquad \int_1^3 dx = 2$$

$$\int_1^3 -x^2\, dx + \int_1^3 4x\, dx - \int_1^3 3\, dx$$

$$-1\int_1^3 x^2\, dx + 4\int_1^3 x\, dx - 3\int_1^3 dx$$

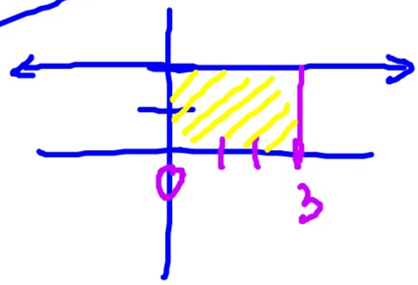
$$\frac{-26}{3} + 4 \cdot 4 - 3 \cdot 2 = \frac{-26}{3} + 10$$

$$= \frac{4}{3}$$

$$\int_0^3 (x^2 + 2)\,dx \qquad \int_0^3 x^2\,dx = 9$$

$$\int_0^3 x^2\,dx + \int_0^3 2\,dx$$

$f(x) = 2$




$$9 + 6$$

$$15$$