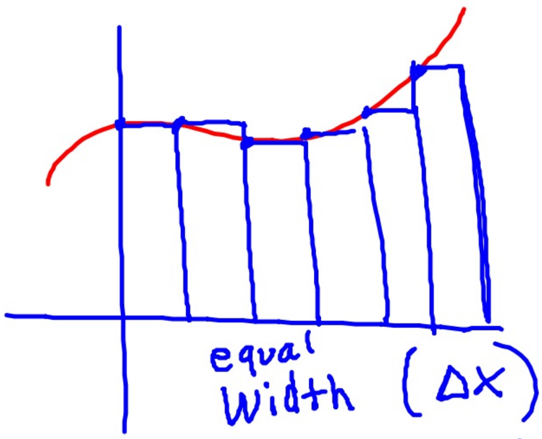
Left and Right Sums: Approximating area under a curve

$$\int_a^b f(x)\,dx$$



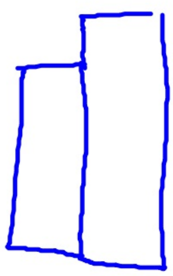
n: number of rectangles

$n \to \infty$; $\Delta x \to 0$

If there is a constant width, the width can be calculated by:

Width = (b - a)/n

$$\Delta x = \frac{b-a}{n}$$



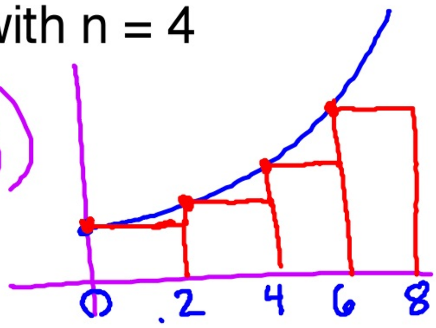
$[7, 19]$ $n = 5$

$$\Delta x = \frac{19-7}{5} = \frac{12}{5}$$

Left sum: Start at the LEFT of the interval $\Delta x = \frac{8-0}{4} = 2$

f(x) = x^2 + 1 on the interval [0, 8] with n = 4

$$\int_0^8 f(x)\,dx \approx 2\left(f(0)+f(2)+f(4)+f(6)\right)$$

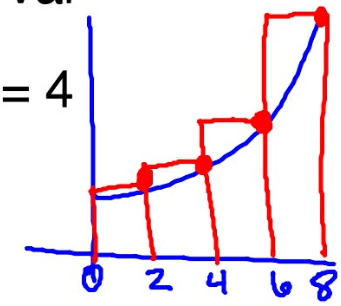


$$\approx 2\left(1+5+17+37\right)$$

$$\approx 120$$

Right Sum: start at the right of the interval

$f(x) = x^2 + 1$ for the interval [0, 8] with n = 4



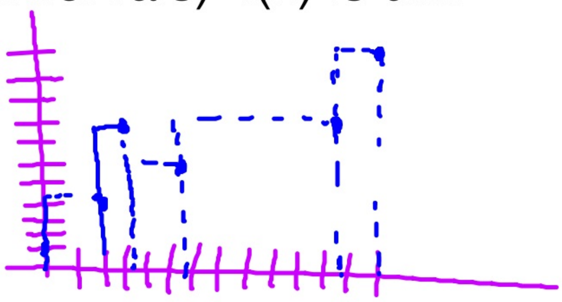
$$\int_0^8 f(x)\,dx \approx 2\left(f(8)+f(6)+f(4)+f(2)\right)$$

$$\approx 2\left(65+37+17+5\right)$$

$$\approx 248$$

Right sum with a chart (5 sub-intervals) f(x) is diff.

X	0	2	3	7	12	13
f(x)	1	4	8	5	8	11



$$\int_0^{13} f(x)\,dx \approx 1\cdot 11 + 5\cdot 8 + 4\cdot 5 + 1\cdot 8 + 2\cdot 4$$

$$\approx 11 + 40 + 20 + 8 + 8$$

$$\approx 87$$

Left sum with a chart (5 sub-intervals) f(x) is diff.

x	0	2	3	7	12	13
f(x)	1	4	8	5	8	11

$$\int_0^{13} f(x)\,dx \approx 2\cdot 1 + 1\cdot 4 + 4\cdot 8 + 5\cdot 5 + 1\cdot 8$$

$$\approx 71$$

WS: 2, 3, 5, 6ab, 7ab, 11, 15