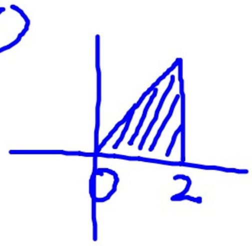
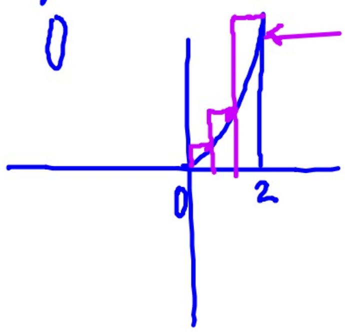
Left, Right, Midpoint and Trapezoidal Sums

Approximating area under a curve

$$\int_0^2 x\,dx$$



$$\int_0^2 x^2\,dx$$



If there is a constant width, the width can be calculated by:

n: number of rectangles

Width = (b - a)/n

$[1, 13]$ $n = 6$

$$W = \frac{b-a}{n} = \frac{13-1}{6} = 2$$

Left sum: Start at the LEFT of the interval

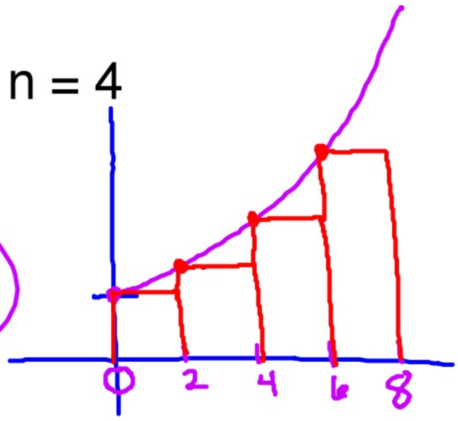
f(x) = x^2 + 1 on the interval [0, 8] with n = 4

$w = 2$

$$\int_0^8 f(x)\,dx \approx 2\left(f(0) + f(2) + f(4) + f(6)\right)$$

$$\approx 2\left(1 + 5 + 17 + 37\right)$$

$$\approx 120$$



Right Sum: start at the right of the interval

$f(x) = x^2 + 1$ for the interval [0, 8] with n = 4

$w = \frac{b-a}{n}$

$W = 2$

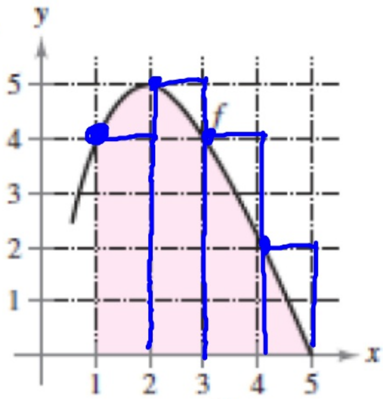
$$\int_0^8 f(x)\,dx \approx 2\big(f(8)+f(6)+f(4)+f(2)\big)$$

$$\approx 2(65+37+17+5)$$

$$\approx 248$$

Estimate the area under the curve.

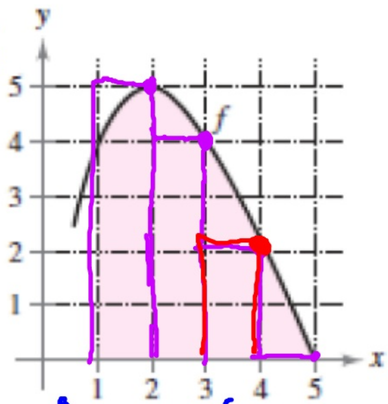
Left



$n=4$

$$\int_1^5 f(x)\,dx \approx 1\left(f(1)+f(2)+f(3)+f(4)\right)$$
$$\approx 1\left(4+5+4+2\right)$$
$$\approx 15$$

Right

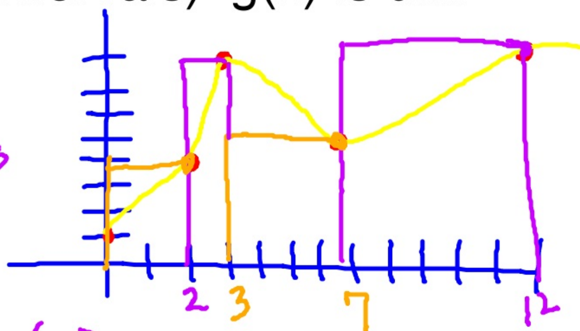


$$\int_1^5 f(x)\,dx \approx 1\left(0+2+4+5\right)$$
$$\approx 11$$

Right sum with a chart (4 sub-intervals) g(x) is diff.

x	0	2	3	7	12
g(x)	1	4	8	5	8

heights



$$\int_0^{12} g(x)\,dx \approx 5(8)+4(5)+1(8)+2(4)$$
$$\approx 40+20+8+8$$
$$\approx 76$$

Left sum with a chart (4 sub-intervals) g(x) is diff.

X	0	2	3	7	12
g(x)	1	4	8	5	8

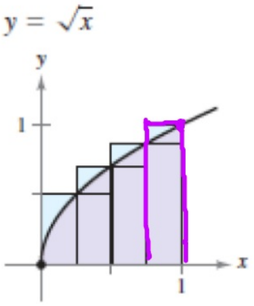
← heights

$$\int_0^{12} g(x)\,dx \approx 2(1) + 1(4) + 4(8) + 5(5)$$

$$\approx 63$$

Right sum with n = 4

$f(x)=\sqrt{x}$



$w=\frac{1-0}{4}$

$w=\frac{1}{4}$

$$\int_0^1 \sqrt{x}\,dx \approx \frac{1}{4}\left(f(1)+f\left(\frac{3}{4}\right)+f\left(\frac{1}{2}\right)+f\left(\frac{1}{4}\right)\right)$$

$$\approx \frac{1}{4}\left(1+\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}+\frac{1}{2}\right)$$

$$\approx \frac{1}{4}\left(\frac{2}{2}+\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}+\frac{1}{2}\right)$$

$$\approx \frac{3+\sqrt{3}+\sqrt{2}}{8}$$

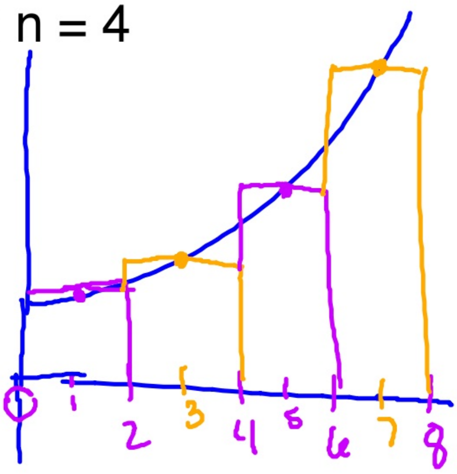
Midpoint Sums: Find the midpoints of the sub-intervals

f(x) = x^2 + 1 on the interval [0, 8] with n = 4

$$\int_0^8 f(x)\,dx \approx 2\left(f(1) + f(3) + f(5) + f(7)\right)$$

$$2\left(2 + 10 + 26 + 50\right)$$

$$\approx 176$$



Midpoint with a chart (3 sub-intervals) $n=3$

x	0	5	10	15	20	25	30
$g(x)$	2	7	5	3	6	8	4

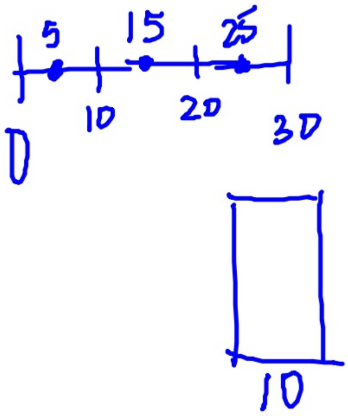
$$W=\frac{b-a}{3}$$

$$W=\frac{30-0}{3}=10$$

$$\int_0^{30} g(x)\,dx \approx 10\big(g(5)+g(15)+g(25)\big)$$

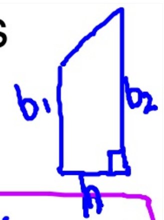
$$\approx 10(7+3+8)$$

$$\approx 180$$



Trapezoids: Approximating the area using trapezoids

f(x) = x^2 +1 on the interval [0, 8] with 4 sub-intervals



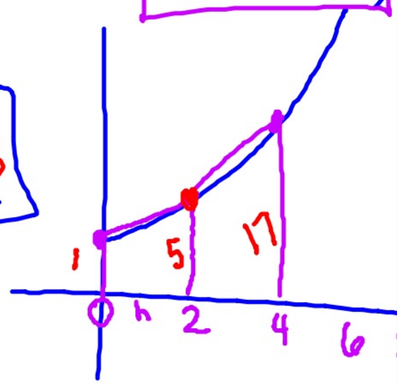
$$\boxed{\frac{1}{2}h(b_1+b_2)}$$

$$\int_0^8 f(x)\,dx \approx$$

$$\frac{1}{2}\cdot 2\left[\underline{1+5} + \underline{5+17} + \underline{17+37} + \underline{37+65}\right]$$

$$1\left[6+22+54+102\right]$$

$$184$$



Trapezoids with a chart (3 sub-intervals)

x	1	3	7	9
$g(x)$	5	8	4	3

$$\frac{1}{2}h(b_1+b_2)$$

$$\frac{1}{2}\left(\_\_\_ + \_\_\_ + \_\_\_\right)$$

$$\int_1^9 g(x)\,dx \approx \frac{1}{2}\left(2(5+8)+4(8+4)+2(4+3)\right)$$

$$\approx \frac{1}{2}\left(26+48+14\right)$$

$$44$$