4.1 Antiderivatives and Indefinite Integration

- Write the general solution of a differential equation.
- Use indefinite integral notation for antiderivatives.
- Use basic integration rules to find antiderivatives.
- Find a particular solution of a differential equation.

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7
49
53 (ignore slope field)
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59
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Differential Equations: finding the general solution or particular solution

General solution

#1 $\frac{dy}{dx} = 2(x - 1)$

$y = 2\left(\frac{x^2}{2} - x\right) + C$

$\boxed{y = x^2 - 2x + C}$

general solution (function)

Particular solution

#2 $\frac{dy}{dx} = x^2 - 1, (-1, 3)$

$y = \frac{1}{3}x^3 - x + C$

find C

$3 = \frac{1}{3}(-1)^3 - (-1) + C$

$3 = -\frac{1}{3} + 1 + C$

$2\frac{1}{3} = C$

$\boxed{y = \frac{1}{3}x^3 - x + 2\frac{1}{3}}$

general

#3

$$\frac{dy}{dx} = 4x - 7$$

$$\boxed{y = 2x^2 - 7x + C}$$

Particular

#4

$$\frac{dy}{dx} = 2x^2 - x + 1 \; ; (0,2)$$

$$y = \frac{2}{3}x^3 - \frac{1}{2}x^2 + x + C$$

$$C = 2$$

$$\boxed{y = \frac{2}{3}x^3 - \frac{1}{2}x^2 + x + 2}$$

Solve the differential equation.

5.) $g'(x) = 3x^2 - 2x + 4 \qquad g(1) = 2$

$g(x) = x^3 - x^2 + 4x + C$

find C

$2 = 1 - 1 + 4 + C$

$-2 = C$

$\boxed{g(x) = x^3 - x^2 + 4x - 2}$

Solve the differential equation (find the original function)

#6 $f''(x) = \sin x,\ f'(0) = 1,\ f(0) = 6$

$f'(x) = -\cos x + C$

$1 = -\cos 0 + C$

$2 = C$

$f'(x) = -\cos x + 2$

$f(x) = -\sin x + 2x + C$

$6 = -\sin 0 + 0 + C$

$6 = C$

$\boxed{f(x) = -\sin x + 2x + 6}$

$$\int (\tan^2 y + 1)\,dy$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\int \sec^2 y\,dy$$

$$\tan y + C$$

37.) $\int (1 - \csc t \cot t)\,dt$

$t + \csc t + C$