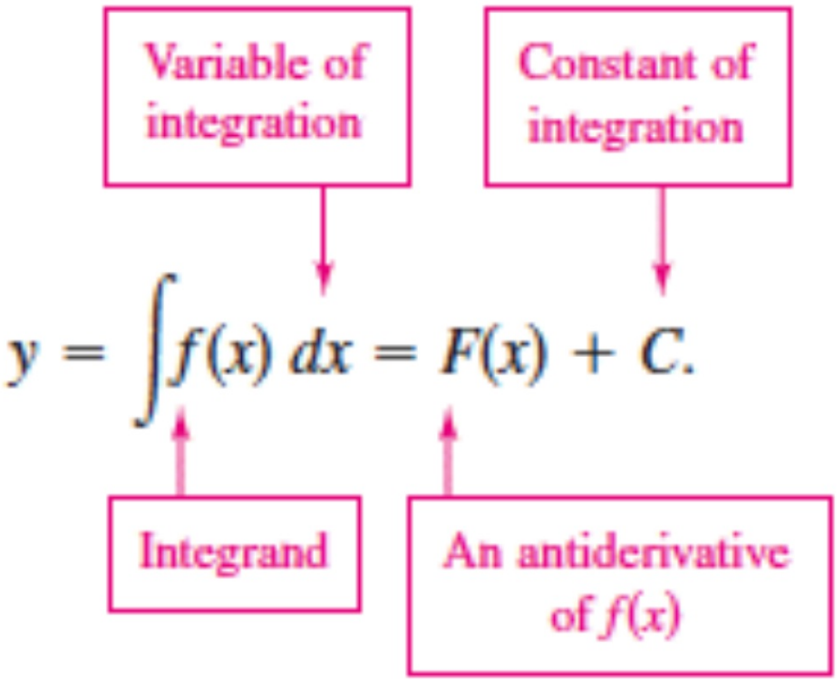
4.1 Antiderivatives and Indefinite Integration

- Write the general solution of a differential equation.
- Use indefinite integral notation for antiderivatives.
- Use basic integration rules to find antiderivatives.
- Find a particular solution of a differential equation.

p. 255
9-43
odd



If you know f'(x), how can you find f(x)?

$$\int 2x\,dx = x^2 + C$$

$f(x) = x^2 + 4$
$f'(x) = 2x$

$f(x) = x^2 - 5$
$f'(x) = 2x$

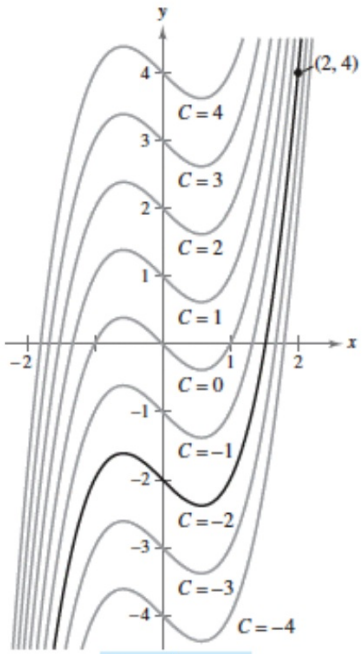
$$\int 7\,dt = 7t + C$$

$f'(x) = 3x^2 - 1$

Integrate to find f(x)

$$\int (3x^2 - 1)\,dx$$

$$x^3 - x + C$$



$F(x) = x^3 - x + C$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

#1 $\int (8x^3 - 9x^2 + 4)\,dx = \frac{8x^4}{4} - \frac{9x^3}{3} + \frac{4x}{1} + C$

$= 2x^4 - 3x^3 + 4x + C$

#2 $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int \left(x^{1/2} + \frac{1}{2}x^{-1/2}\right) dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2}\cdot\frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

#3 $\int \frac{x^2 + 2x - 3}{x^4}\,dx = \int \frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4}\,dx$

$= \int x^{-2} + 2x^{-3} - 3x^{-4}\,dx = \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3}$

$-x^{-1} - x^{-2} + x^{-3} + C$

#4 $\int (2t^2 - 1)^2\,dt = \int (4t^4 - 4t^2 + 1)\,dt$

$\frac{4t^5}{5} - \frac{4t^3}{3} + t + C$

Integration Rules

Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

secxsecxtanx

-cscxcscxcotx

#5 $\int \sec y(\tan y - \sec y)\, dy$

$\int (\sec y \tan y - \sec^2 y)\, dy = \sec y - \tan y + C$

#6 $\int \frac{\sin x}{1 - \sin^2 x}\, dx$

think Pythagorean!

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\int \frac{\sin x}{\cos^2 x}\, dx = \int \frac{\sin x \cdot 1}{\cos x \cos x}\, dx$

$= \int \tan x \sec x\, dx = \sec x + C$